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Manual of Codes of Practice for the Determination of Uncertainties in Mechanical Tests on Metallic Materials

Code of Practice No. 01

The Determination of Uncertainties in High Cycle Fatigue Testing (for plain and notch-sensitive specimens)

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CONTENTS

1. SCOPE

This procedure covers the determination of uncertainties in axial force controlled fatigue tests. This procedure also determines the uncertainties of the notch sensitivity factor. The procedure is restricted to testing at constant amplitude in air and at room temperature.

2. SYMBOLS AND DEFINITIONS

For a complete list of symbols and definitions of terms on uncertainties, see Reference 1, Section 2. The following are the symbols and definitions used in this procedure.

A	number function of N, used in staircase test (for reference: UNI 3964)
c_i	sensitivity coefficient
CoP	Code of Practice
d	stair case step level
D_n	nominal diameter
D_l	diameter of the specimen at the notch (local)
D_n/D_l	ratio of D_n and D_1
d_v	divisor used to calculate the standard uncertainty
F or M	force or bending
k	coverage factor used to calculate expanded uncertainty
k	slope of the characteristic Log σ - Log N
K_f	fatigue notch factor
K_t	stress concentration factor
N	total of less frequent events
N_c	number of cycles
n	number of repeat measurements
p	confidence level
q	random variable
$\bar{q}$	arithmetic mean of the values of the random variable q
r	radius of the notch
r/D_l	ratio of r and D_1
S	experimental standard deviation (of a random variable) determined from a limited number of measurements, n
$S_{A,Kt>1}$	fatigue limit of notched specimen
$S_{A,Kt=1}$	fatigue limit of straight specimen
S_n	nominal cross section
S_l	local cross section
s_0	lower stress
s_n	nominal stress
s_{max}	local stress
Q	notch sensitivity

V	displayed value or mean computed value
U	expanded uncertainty associated to V
X	confidence level

3. INTRODUCTION

It is good practice in any measurement to evaluate and report the uncertainty associated with the test results. A statement of uncertainty may be required by a customer who wishes to know the limits within which the reported result may be assumed to lie, or the test laboratory itself may wish to develop a better understanding of which particular aspects of the test procedure have the greatest effect on results so that this may be monitored more closely. This Code of Practice (CoP) has been prepared within UNCERT, a project funded by the European Commission's Standards, Measurement and Testing programme under reference SMT4-CT97-2165 to simplify the way in which uncertainties are evaluated.

The aim is to produce a series of documents in a common format that is easily understood and accessible to customers, test laboratories and accreditation authorities.

This CoP is one of seventeen produced by the UNCERT consortium for the estimation of uncertainties associated with mechanical tests on metallic materials. Reference 1 is divided into six sections as follows, with all the individual CoPs included in Section 6.

1. Introduction to the evaluation of uncertainty
2. Glossary of definitions and symbols
3. Typical sources of uncertainty in materials testing
4. Guidelines for estimation of uncertainty for a test series
5. Guidelines for reporting uncertainty
6. Individual Codes of Practice (of which this is one) for the estimation of uncertainties in mechanical tests on metallic materials.

This CoP can be used as a stand-alone document. For further background information on the measurement uncertainty and values of standard uncertainties of the equipment and instrumentation used commonly in material testing, the user may need to refer to Section 3 in Reference 1. The individual CoPs are kept as simple as possible by following the same structure; viz:

- The main procedure
- Quantifying the major contributions to the uncertainty for that test type (Appendix A)
- A worked example (Appendix B)

This CoP guides the user through the various steps to be carried out in order to estimate the uncertainty in Notch Sensitivity and Fatigue Testing.

4. A PROCEDURE FOR THE ESTIMATION OF UNCERTAINTIES IN FATIGUE TEST AND NOTCH SENSITIVITY

The objective of this procedure is to evaluate the uncertainty of each calculated quantity in a fatigue test with a given confidence level. It is assumed throughout the procedure that the test has been performed and the raw data available. The final value will thus be presented in the following way:

$$V \pm U \text{ with a confidence level of } X\%$$

Where, V is the displayed value or mean computed value
U is the expanded uncertainty associated to V
X is the confidence level.

This document guides the user through six steps to determine the above values.
Before starting, the user must be aware of the following:

1. The relevant standard is ASTM E466 - 96.
2. The quantities than are to be evaluated and produced as test results.
3. The testing procedure followed during the test.
4. The testing apparatus' specifications and/or calibration certificates.
5. The raw data gathered during the test.
6. The required confidence level for each desired quantity (for most applications, a confidence level of 95% will be retained as default value).

This general process permits the statistical influence of each source of uncertainty in the final result to be tabulated. The following sections detail the six steps of this process.

Step 1 – Identifying the Parameters for Which Uncertainty is to be Estimated.

The first Step consists of setting the quantities that are to be determined as result of the test:

1. Calculation of the uncertainty of the fatigue limit.
2. Calculation of the uncertainty of the number of cycles.
3. Calculation of the uncertainty of the stress concentration factor.
4. Calculation of the uncertainty of the fatigue notch factor.
5. Calculation of the uncertainty of the notch sensitivity.

It is useful to construct up a table of all quantities evaluated during the test. An example of this is shown in Table 1. Part A contains all terms measured during an axial fatigue test, Part B contains all terms calculated, whilst all invariant values are shown in Part C. By changing the procedure this list may vary.

Table 1. Measured, Calculated and Invariant Quantities

	Measurements	Unit of measurement	Symbol
Part A	Nominal diameter Diameter of the specimen at the notch (local)	[mm] [mm]	D_n D_l
	Force or bending Radius of the notch	[N] or [N mm] [mm]	F or M R
Part B	Stress concentration factor	/	K_t
	Fatigue notch factor	/	K_f
	Notch sensitivity	/	Q
	r/D_l ratio	/	r/D_l
	D_n/D_l ratio Nominal cross section Local cross section Number of cycles Stair case step level Total of less frequent events Parameter (for reference: UNI 3964) Lower stress Nominal stress Fatigue limit of notched specimen Local stress Fatigue limit of the un-notched specimen	/ [mm^2] [mm^2] [cycles] [N/mm^2] / / [N/mm^2] [N/mm^2] [N/mm^2] [N/mm^2] [N/mm^2]	D_n / D_l S_n S_l N_c D N A σ_0 σ_n $S_{A,\ Kt>1}$ σ_{max} $S_{A,\ Kt=1}$
Part C	Slope of the characteristic Log σ - Log N	/	K

Step 2 – Identifying all Sources of Uncertainty in a Test

Table 2 lists possible sources of uncertainty divided into five distinct categories. It is important to note that the list is NOT exhaustive. Many further sources may be numbered depending on specific testing configurations. The user is strongly advised to draft their own list corresponding to their test facilities.

Table 2. Examples of Sources of Uncertainty

Category	Example of Sources of Uncertainty	Importance of the Sources
Apparatus	Load cell calibration	Influential *
	Load cell sensitivity	Influential *
	Tooling alignment	Influential *
	Dynamic control of load	Influential *
	Drift of static load	Influential *
Method	Specimen failure criteria	Not influential
Environment	Temperature	Not influential
Operator	Selection of results for the calculation	Not applicable
Test Piece	Original diameter	Influential
	Notched diameter	Influential
	Nominal diameter	Influential
	Radius of the notch	Influential

** no influence on the stress concentration factor*

Step 3 – Classifying the Uncertainty According to Type A or B

In accordance with ISO TAG 4 'Guide to the Expression of Uncertainties in Measurement' (Paragraph 0.7), sources of uncertainty can be classified as Type A or B. This classification is dependent on the way their influence is quantified. If a source's influence is evaluated by statistical means (ie on a number of repeated observations), it is classified Type A. If a source's influence is evaluated by any other means (may for example, manufacturer's documents, certification), it is classified Type B.

It should be noted that a source may be classified as Type A or B depending on the way it is estimated. For example, if the diameter of a cylindrical specimen is measured once, that parameter is considered Type B. If the mean value of ten consecutive measurements is taken, then the parameter is Type A.

When a computation table is first drafted, most sources will be Type B (quantified by reference to documentation or estimation). As experience builds, more and more sources may be quantified Type A, thus reducing the overall uncertainty.

The sources of uncertainty identified in Step 2 are classified as Type A or B in Table 3.

Table 3. Classification of the Sources

Category	Example of Sources of Uncertainty	Classification of the Types
Apparatus	Load Cell calibration	B
	Load Cell sensitivity	B
	Tooling alignment	B
	Dynamic control of load	B
	Drift of static load	B
	Test frequency	Not influential
Method	Specimen failure criteria	Not influential
Environment	Temperature	Not influential
Operator	Selection of results for the calculation	Not applicable
Test – Piece	Original diameter	A, B
	Notched diameter	A, B
	Nominal diameter	A, B
	Radius of the notch	A, B

Step 4 – Estimating the Standard Uncertainty for each Source of Uncertainty

Sources are classified as Type A or B as each type has its own method of quantification.

By definition, a Type A source of uncertainty is already a product of statistical computation. Type A requires calculation of the average (x_m) of a series of measurements; it also requires the calculation of the standard deviation (s) and the standard uncertainty (S). In this case it is necessary to establish if the calculation of standard deviation is completed by considering all the population or not. This procedure utilises the data as a sample of the population.

The Type B source of uncertainty can have various origins: a manufacturer's indication, a certification, an expert's estimation or any other mean of evaluation. For type B sources, it is necessary for the user to estimate the most appropriate (most probable) distribution for each source; when this has been chosen the user divides the standard uncertainty by using the divisor d_v.

TYPE A: Standard uncertainty S.
TYPE B: Standard uncertainty u (x) = d_v u .

Table 4. Correction Factor k' According to the Estimated Distribution

Category	**Measurand**			**Uncertainties**		
Source of Uncertainty	**Measurand Affected**	**Nominal or Averaged Value**	**Type**	**Probabl. Distrib.**	**Divisor** d_v	$u(x_i)$
Apparatus						
Load cell calibration	F	(N)	B	Rectangular	$1/\sqrt{3}$	u (F)
Load cell sensitivity	F	(N)	B	Rectangular	$1/\sqrt{3}$	u (F)
Tooling alignment	F	(N)	B	Rectangular	$1/\sqrt{3}$	u (F)
Dynamic control of load	F	(N)	B	Rectangular	$1/\sqrt{3}$	u (F)
Drift of static load	F	(N)	B	Rectangular	$1/\sqrt{3}$	u (F)
Test frequency			n.i.			
Method						
Specimen failure criteria			n.a.			
Environment						
Temperature			n.i.			
Operator						
Selection of results for the calculation			n.a.			
Test – Piece						
Original cross section			n.a.			
Original diameter	D	(mm)	A , B	Rectangular	$1/\sqrt{3}$	u(D)
Notched diameter	D_l	(mm)	A , B	Rectangular	$1/\sqrt{3}$	u(D_l)
Nominal diameter	D_n	(mm)	A , B	Rectangular	$1/\sqrt{3}$	u(D_n)
Radius of the notch	r	(mm)	A , B	Rectangular	$1/\sqrt{3}$	u(r)

1) Appendix A describes the technical background to the combined standard uncertainty of the fatigue limit of a notched specimen (I), the combined standard uncertainty of the stress concentration factor (II), the combined standard uncertainty of the fatigue notch factor (III) and the combined standard uncertainty of the notch sensitivity (IV).

2) Appendix B contains a worked example for calculating uncertainties in Notch sensitivity.

Examples of a possible solution are contained in the end pages of Appendix B.

Step 5 – Computing the Combined Uncertainty u_c

Once each source of uncertainty is estimated, it is possible to calculate the combined standard uncertainty $u_c(x)$ and/or the relative combined standard uncertainty $u_c(x)/x$. These uncertainties correspond to plus or minus one standard deviation on the normal law. This represents the studied quantities distribution. This law takes into account all estimated sources as if they were fully independent in the following way:

Combined standard uncertainty: $(u_c)^2 = (\sum S^2 + \sum u^2)$

It is possible, for Type A, when it has been calculated the standard uncertainty S, to calculate the relative standard uncertainty by dividing for the average: $u_c(x)/ x = S / x_m$. For Type B when it has been obtained the standard uncertainty, this can be transformed in a relative standard uncertainty by dividing for the measured value x: $u_c(x)/ x$.

If the relative standard uncertainty is directly calculated the right formula is:

$$[u_c (x)/x]^2 = \Sigma^N_{i=1} [c_i\, u(x_i) / x_i]^2$$

where, c_i = sensitivity coefficient

The following tables list the sensitivity coefficients c_i of each source of uncertainty necessary for the calculation of the relative combined standard uncertainty of the fatigue limit (Table 5), of the number of cycles (Table 6), of k_t (Table 7), of k_f (Table 8), of q (Table 9). The coefficients are described in Appendix A.

Table 5. Sensitivity Coefficients for the Calculation of the Relative Combined Standard Uncertainty of the Fatigue Limit

Sources of Uncertainty	**Influence Coefficients**
Load Cell calibration	1
Load Cell sensitivity	1
Tooling alignment	1
Dynamic control of load	1
Drift of static load	1
Original cross section	-1
Original diameter	2
Sentitivity of the instrument	2

Table 6. Sensitivity Influence Coefficients for the Calculation of the Relative Combined Standard Uncertainty of the Number of Cycles (N_2).

Sources of Uncertainty	**Sensitivity Coefficients**
Stress in point 1 σ_1	+ k
Stress in point 2 σ_2	- k
Number of cycles of stress 1 N_1	1
Slope k of the characteristic Logσ - LogN	not applicable

Table 7. Sensitivity Coefficients for the Calculation of the Relative Combined Standard Uncertainty of the Stress Concentration Factor (K_t).

Sources of Uncertainty	**Sensitivity Coefficients**
Maximum stress, σ_{1max}	1
Nominal stress, σ_n	-1

Table 8. Sensitivity Coefficients for the Calculation of the Relative Combined Standard Uncertainty of the Notch Factor (K_f).

Sources of Uncertainty	**Sensitivity Coefficients**
Fatigue limit of un-notched specimen, $S_{A, Kt=1}$	1
Fatigue limit of notched specimen, $S_{A, Kt>1}$	-1

Table 9. Sensitivity Coefficients for the Calculation of the Relative Combined Standard Uncertainty of the Notch Sensitivity (Q).

Sources of Uncertainty	**Sensitivity Coefficients**
k_f	1
k_t	-1

Step 6 – Computing the Expanded Uncertainty U

The final Step is optional and depends on the customer's requirements. The expanded uncertainty U is broader than the combined standard uncertainty; the confidence level associated with it is also greater. The combined standard uncertainty u_c has a confidence level of 68.27% corresponding to plus or minus one standard deviation. Where a high confidence level is needed (for example, aerospace industry, electronics), the combined standard uncertainty u_c is multiplied by a coverage factor k for obtaining the expanded uncertainty U. If u_c is, for example, tripled the corresponding confidence level is 99.73%.

Table 10 gathers some coverage factors k leading to an X% confidence level.

Table 10. Coverage Factor According to Requested Confidence Level

Confidence level: X%	Coverage factor k
68.27	1
90	1.645
95	1.960
95.45	2
99	2.576
99.73	3

Step 7 – Reporting of Results

Once the expanded standard uncertainty has been computed, the final result can be given as:

$$V = y \pm U \text{ with a confidence level of } X\%$$

Where, V is the estimated value of the measurand
y is the test (or measurement) mean result
U is the expanded uncertainty associated with y

5. REFERENCES

1. *Manual of Codes of Practice for the determination of uncertainties in mechanical tests on metallic materials*. Project UNCERT, EU Contract SMT4-CT97-2165, Standards Measurement & Testing Programme, ISBN 0-946754-41-1, Issue 1, September 2000.

2. BIPM, IEC, IFCC, ISO, IUPAC, OIML, *Guide to the expression of Uncertainty in Measurement.* International Organisation for Standardisation, Geneva, Switzerland, ISBN 92-67-10188-9, First Edition, 1993. [This Guide is often referred to as the GUM or the ISO TAG4 document after the ISO Technical Advisory Group that drafted it.]

3. ASTM Standard, *Conducting Force Controlled Constant Amplitude Axial Fatigue Tests of Metallic Materials***.** ASTM E 466 – 96.

4. UNI Standard, *Mechanical Testing of Metallic Materials Fatigue Testing at Room Temperature.* UNI 3964 – 85.

APPENDIX A

A1 - Uncertainty of the fatigue limit of an un-notched specimen

This appendix is designed for the simple explanation of the formulas necessary to calculate combined standard uncertainty of the fatigue limit σ_D, $u_c(\sigma_D)$, of an un-notched specimen, in accordance with ISO/IEC "*Expression of Uncertainty: 1992*".

The formulas used for the calculation of σ_D are in accordance with the document: *"UNI 3964 Prove Meccaniche dei Materiali Metallici - Prove di Fatica a Temperatura Ambiente"*.

Axial Fatigue Limit σ_D

TERMINOLOGY
σ_D = fatigue limit;
σ_0 = lowest stress;
d = stair - case step stress;
N_e = total of less frequent events;
A = number function of N;
S = cross section;
F = force - axial test.

PART I

The formula for the calculation of σ_D is:

$$\sigma_{D(68.5\%)} = \sigma_0 + d\,(A/N_e \pm 0.5)$$

The general combined standard uncertainty $u_c(Y)$ is expressed by:

$[u_c(Y)]^2 = \Sigma\,(\partial f/\partial X_i)^2\,u^2(X_i)$. Using this formula the combined standard uncertainty $u_c(\sigma_D)$ is:

$$u_c(\sigma_D) = (\partial\sigma_D/\partial\sigma_0)^2\,\underline{u^2(\sigma_0)} + (\partial\sigma_D/\partial d)^2\,\underline{u^2(d)}$$

$$(\partial\sigma_D/\partial\sigma_0) = 1$$
$$(\partial\sigma_D/\partial d) = (A/N_e \pm 0.5)$$

The underlined term should be analysed in detail.

PART II

The aim of part II is the calculation of the combined standard uncertainty of σ_0, whose symbol is $u_c(\sigma_0)$.

$$\rightarrow \sigma_0 = F_0 / S \quad [\text{MPa}]$$

$$u^2_c(\sigma_0) = (\partial\sigma_0/\partial F_0)^2 \, \mathbf{u^2_F (F_0)} + (\partial\sigma_0/\partial S)^2 \, \underline{u^2_S (S)}$$

$$(\partial\sigma_0/\partial F_0) = 1 / S$$
$$(\partial\sigma_0/\partial S) = - F_0 / S^2$$

The bold face terms do not require any further calculation while the underlined term should be analysed further.

PART III

The aim of part III is the calculation of the combined standard uncertainty of the cross section S, whose symbol is $u_c(S)$.

$$\rightarrow S = \pi (D/2)^2 = \pi D^2 /4$$

$$[u_c(S)]^2 = (\partial S/\partial D)^2 \, \mathbf{u^2(D)}$$
$$(\partial S/\partial D) = \pi D / 2$$

PART IV

The aim of part IV is the calculation of the combined standard uncertainty of d, whose symbol is $u_c(d)$.

$$\rightarrow d = F_d / S \quad [\text{MPa}]$$

$$u^2_c(d) = (\partial d/\partial F_d)^2 \, \mathbf{u^2_F (F_d)} + (\partial d_l/\partial S)^2 \, \underline{u^2_S (S)}$$

$$(\partial d/\partial F_d) = 1 / S$$
$$(\partial d/\partial S) = - F_d / S^2$$

Although these are the correct formulas for obtaining the uncertainty of d, experience suggests that this uncertainty is equal to u(D).

CONCLUSIONS

The global formula necessary for the calculation of combined standard uncertainty of σ_D is:

$$u^2_c(\sigma_D) = [\,(1 / S)^2 \, \mathbf{u^2_F (F_0)} + (- F_0 / S^2)^2 (\pi D/2)^2 \, \mathbf{u^2(D)}]\,[1+ (A/N_e \pm 0.5)^2] \quad \text{I.1}$$

The bold face terms do not require any further calculation but can be valued by the operator using Table I-A and Table I-B enclosed to the procedure. A numerical example is also contained in the following pages.

A2 - Uncertainty of the Fatigue limit of a notched specimen

This appendix is designed for the simple explanation of the formulas necessary to calculate combined standard uncertainty of the Fatigue limit σ_D, $u_c(\sigma_D)$, of a notched specimen, in accordance with ISO/IEC "*Expression of Uncertainty: 1992*".

The formulas used for the calculation of σ_D are in accordance with the document: "*UNI 3964 Prove Meccaniche dei Materiali Metallici - Prove di Fatica a Temperatura Ambiente*".

Axial fatigue limit s $_D$ of a notched specimen

TERMINOLOGY
$\sigma_D = S_{A,\,kt>1}$ = Fatigue limit;
σ_0 = lowest stress;
d = stair - case step stress;
N = total of less frequent events;
A = parameter function of N;
S_l = local cross section;
F = force - axial test.

<u>*PART I*</u>

The formula for the calculation of σ_D is:

$$\sigma_{D(50\%)} = \sigma_0 + d\,(A/N_e \pm 0.5)$$

The general combined standard uncertainty $u_c(Y)$ is expressed by:

$[u_c(Y)]^2 = \Sigma\,(\partial f/\partial X_i)^2\,u^2(X_i)$. Using this formula the combined standard uncertainty $u_c(\sigma_D)$ is:

$$u_c(\sigma_D) = (\partial\sigma_D/\partial\sigma_0)^2\,\underline{u^2(\sigma_0)} + (\partial\sigma_D/\partial d)^2\,\underline{u^2(d)}$$

$$(\partial\sigma_D/\partial\sigma_0) = 1$$
$$(\partial\sigma_D/\partial d) = (A/N_e \pm 0.5)$$

The underlined term should be carefully calculated by the laboratory.

<u>*PART II*</u>

The aim of part II is the calculation of the combined standard uncertainty of σ_0, whose symbol is $u_c(\sigma_0)$.

$$\rightarrow \sigma_0 = F_0 / S_l \quad [\text{MPa}]$$

$$u^2_c\,(\sigma_0) = (\partial\sigma_{0l}/\partial F_0)^2\,\mathbf{u^2_F\,(F_0)} + (\partial\sigma_{0l}/\partial S)^2\,\underline{u^2_S\,(S_l)}$$

$$(\partial\sigma_0/\partial F_0) = 1 / S_l$$
$$(\partial\sigma_0/\partial S_l) = - F_0 / S^2_1$$

The bold face terms do not need any further calculation, while the underlined term requires further analysis.

PART III

The aim of part III is the calculation of the combined standard uncertainty of the cross section S_l, whose symbol is $u_c(S_l)$.

$$\rightarrow S_l = \pi (D_l / 2)^2 = \pi D^2_1 /4$$

$$[u_c(S_l)]^2 = (\partial S/\partial D_l)^2 \mathbf{u^2(D_l)}$$
$$(\partial S/\partial D_l) = \pi D_l / 2$$

PART IV

The aim of this part is the calculation of the combined standard uncertainty of d, whose symbol is $u_c(d)$.

$$\rightarrow d = F_d / S_l \quad [MPa]$$

$$u^2_c (d) = (\partial d/\partial F_d)^2 \mathbf{u^2_F (F_d)} + (\partial d_l/\partial S)^2 \underline{u^2_S (S_l)}$$

$$(\partial d/\partial F_d) = 1 / S_l$$
$$(\partial d/\partial S_l) = - F_d / S^2_1$$

Although these are the right formulas for obtaining the uncertainty of d, experience teaches this uncertainty is equal to u(D).

CONCLUSIONS

The global formula necessary for the calculus of combined standard uncertainty of σ_D is:

$$u_c^2(\sigma_D) = [(1 / S_l)^2 \mathbf{u^2_F (F_0)} + (- F_0 / S^2_1)^2 (\pi D_l/2)^2 \mathbf{u^2(D_l)}] \times [1+ (A/N_e \pm 0.5)^2] \quad II.1$$

The bold face terms do not require any further calculation but can be valued by the operator using Table A and Table B enclosed to the procedure. A numerical example is also contained in the following pages.

A3 - Uncertainty of the number of cycles

This appendix is designed for the simple explanation of the formulas necessary to calculate relative combined standard uncertainty of the number of cycles N, $u_c(N)/N$, in accordance with ISO/IEC "*Expression of Uncertainty: 1992*".

TERMINOLOGY
N = number of cycles;
N_A = number of cycles in a point A;
σ = stress;
σ_A = stress of point A;
k = characteristic slope of the S-N curve (see table for the choice of its value);
S = cross section;
D = diameter.

The calculation of the number of cycles, corresponding to an assigned tension σ, is:

$$N = N_A \; (\sigma / \sigma_A)^{-k} \quad [cycles]$$

The general relative combined standard uncertainty $[u_c(Y)/Y]$ is expressed by:

$[u_c(Y)/Y]^2 = \Sigma \; [c_i \; u(X_i)/X_i]^2$ whit c_i influence coefficient of X_i. Using this formula the relative combined standard uncertainty $u_c(N)/N$ is:

$$[u_c(N)/N]^2 = [-k * \mathbf{u(s)/s}\;]^2 + [k * \mathbf{u(s_A)/s_A}\;]^2 + [\;\mathbf{u(N_A)/N_A}\;]^2 \qquad \text{III.1}$$

The calculation of the relative combined standard uncertainty of the stresses σ, $u_c(\sigma)/\sigma$, and of σ_A, $u_c(\sigma_A)/\sigma_A$ are shown in *Appendix I.*

Slope k in many cases is the characteristic of the class of material; using developed procedure (of USB - Unified Scatter Band) it is possible to determine this important constant for the S - N curve.

Although it is possible to obtain the uncertainty of k, this procedure doesn't calculate it as its value depends on the method chosen for the calculation of k itself, (for example k can be valued by algebraic equations for first order regression calculations).

A4 - Uncertainty of the stress concentration factor

This appendix is designed for the simple explanation of the formulas necessary to calculate combined standard uncertainty of the stress concentration factor k_t, $u_c(k_t)$, in accordance with ISO/IEC "*Expression of Uncertainty: 1992*".

TERMINOLOGY
σ_{max} = local stress;
σ_n =nominal stress;
r = radius of the notch;
D_l =diameter of the specimen at the notch;
D_n = Nominal diameter;
K_t = stress concentration factor.

The formula for the calculation of the stress concentration factor is $K_t = \sigma_{max} / \sigma_n$.

Generally the value of k_t is obtained by (r/D_l; K_t) graphics where K_t is function of (r/D_l; D_n/D_l). The necessary inputs for the right reading of K_t on these graphics are:

* specimen's shape;
* loads;
* r/D_l;
* D_n/D_l.

This procedure makes the hypothesis that the ratio D_n/D_l is constant. The user can calculate this ratio using nominal values. When it is known which is the right graphic to use, the first step is the calculation of the relative combined standard uncertainty of (r/D_l).

<u>*PART I*</u>

The general relative combined standard uncertainty $[u_c(Y)/Y]$ is expressed by:

$[u_c(Y)/Y]^2 = \Sigma\ [c_i\ u(X_i)/X_i]^2$ with c_i influence coefficient of X_i. Using this formula the relative combined standard uncertainty $u_c(r/D_l)/ r/D_l$ is:

$$[u_c(r/D_l)/ r/D_l]^2 = [1 * \mathbf{u(r)/r}\ ]^2 + [-1 * \mathbf{u(D_l)\ /\ D_l}\ ]^2$$

An example of this calculation is contained in the following pages.

<u>*PART II*</u>

Once the relative combined standard uncertainty is known, it is possible to obtain the combined standard uncertainty = $[u_c(r/D_l)/ r/D_l] * r/D_l$.

PART III

On the plane (K_t - r/D_l), at the abscissa r/D_l on the graphic D_n/D_l = constant, it is possible to draw a tangent to the curve with equation:

$$K_t = - a (r/D_l) + b.$$

The user calculates constants *a* and *b* and with the following formulas also k_t's uncertainty:

$$u^2_c(K_t) = (\partial K_t/\partial r)^2 \mathbf{u^2(r)} + (\partial K_t/\partial D_l)^2 \mathbf{u^2(D_l)}$$

$$(\partial K_t /\partial r) = -a / D_l$$

$$(\partial K_t /\partial D_l) = a / D^2_l$$

In conclusion the formula is: $u^2_c(K_t) = (-a / D_l)^2 \mathbf{u^2(r)} + (a / D^2_l)^2 \mathbf{u^2(D_l)}$

An example is contained in the following pages.

A5 - Uncertainty of the fatigue notch factor

This appendix is designed for the simple explanation of the formulas necessary to calculate relative combined standard uncertainty of the Fatigue notched factor $k_{f,}$ $u_c(k_f)$, in accordance with ISO/IEC "*Expression of Uncertainty: 1992*".

TERMINOLOGY
K_f = fatigue notch factor;
$S_{A,\ Kt=1}$ = fatigue limit of an un-notched specimen (see Procedure ***Appendix A "Fatigue test"***);
$S_{A,\ Kt>1}$ = fatigue limit of a notched specimen (see ***Appendix A***);

The formula for the calculation of the fatigue notched factor is:

$$K_f = S_{A,\ Kt=1} / S_{A,\ Kt>1}$$

The general relative combined standard uncertainty $u_c(Y)/Y$ is expressed by:

$$[u_c(Y)/Y]^2 = \Sigma\ [c_i\, u(X_i)/X_i]^2 \text{ whit } c_i \text{ influence coefficient of } X_i.$$

Using this formula the relative combined standard uncertainty $u_c (K_f)/K_t$ is:

$$[u_c(K_f)/K_f]^2 = [1 * \mathbf{u(S_{A,\ Kt=1})/\ S_{A,\ Kt=1}]^2} + [-1 * \mathbf{u(S_{A,\ Kt>1})/\ S_{A,\ Kt>1}]^2}$$

An example of this calculation is contained in the following pages.

A6 - Uncertainty of the notch sensitivity

This appendix is designed for the simple explanation of the formulas necessary to calculate combined standard uncertainty of the notch sensitivity q $u_c(q)$, in accordance with ISO/IEC *"Expression of Uncertainty: 1992"*.

TERMINOLOGY
K_f = fatigue notch factor;
K_t = stress concentration factor;
Q = notch sensitivity;

The formula for the calculation of the notch sensitivity is:

$$Q = (K_f - 1) / (K_t - 1)$$

The general combined standard uncertainty $u_c(Y)$ is expressed by:

$[u_c(Y)]^2 = \Sigma\ (\partial f/\partial X_i)^2\ u^2(X_i)$. Using this formula the combined standard uncertainty $u_c\ (Q)$ is:

$$u^2{}_c(Q) = (\partial Q/\partial K_f)^2\ \mathbf{u^2(K_f)} + (\partial Q/\ \partial K_t)^2\ \mathbf{u^2(K_t)}$$

$$(\partial Q/\partial K_f) = 1/\ (K_t - 1)$$
$$(\partial Q/\ \partial K_t) = (-K_f + 1)\ /\ (K_t - 1)^2$$

In conclusion:

$$u^2{}_c(Q) = [1/\ (K_t - 1)]^2\ \mathbf{u^2(K_f)} + [\ (-K_f + 1)\ /\ (K_t - 1)^2]^2\ \mathbf{u^2(K_t)}$$

APPENDIX B

EXAMPLE 1

The example is about the calculation of the fatigue limit's uncertainty, when the fatigue limit is obtained by a stair - case test.

Equation I.1 of ***Appendix I*** gives the uncertainty requested:

$$u^2_c(\sigma_D) = [\ (1 / S)^2\ \mathbf{u^2_F\ (F_0)} + (- F_0 / S^2)^2 (\pi\ D/2)^2\ \mathbf{u^2(D)}]\ [1 + (A/N_e \pm 0.5)^2] \quad \text{I.1}$$

DATA
The specimen's nominal diameter is 8.00 mm;
alternative max. load F_0 is 2764.6 N;
A = 6;
N_e = 5;
$(A/N_e + 0.5) = 1.7$
Cross section S:
$S = \pi\ (D/2)^2 = 50.26548\ [mm^2]$.
$\sigma_D = 58.5\ [N/mm^2]$ this is the experimental result from equation $\sigma_{D(68.5\%)} = \sigma_0 + d\ (A/N_e \pm 0.5)$.

$$u^2_c(\sigma_D) = [\ (1 / S)^2\ \mathbf{u^2_F\ (F_0)} + (- F_0 / S^2)^2 (\pi\ D/2)^2\ \mathbf{u^2(D)}]\ [\ 1 + 1.7^2] \quad \text{I.2}$$

Table I - A is designed for simply obtaining the combined standard uncertainty of the *Diameter D*, $\mathbf{u_c(D)}$. The all values can vary in accordance with each test method and apparatus; the user can use this table remembering to vary each term if necessary.

Table I - B is designed for simply obtaining the combined standard uncertainty of the *Load F*, $\mathbf{u_c(F_0)}$. All values can vary in accordance with each test method and apparatus; the user can use this table remembering to vary each term if necessary.

It has been obtained:
combined standard uncertainty $\mathbf{u_c(D)}$ = 0.00879 [mm];
combined standard uncertainty $\mathbf{u_c(F)}$ = 17.2896 [N].

It is now possible to calculate the combined standard uncertainty of total stress σ_{total}:

$$u^2_c\ (\sigma_D) = [(1 /50.27)^2\ (17.2896)^2 + (- 2764.6 / 50.27^2)^2 (\ \pi\ 8.0/2)^2\ (0.00879)^2] * [1+ 1.7^2]$$

$$u^2_c(\sigma_D) = 0.13292 * [1 + 1.7^2] = 0.5170588$$

$$u_c\ (\sigma_D) = 0.7191\ [N/mm^2]$$

The uncertainty calculated corresponds to 68.5% probability of survival, that means a coverage factor k = 1.

We prefer to give the 95% probability of survival, with a coverage factor k = 2, so it is necessary to obtain the expanded standard uncertainty:

$$U(\sigma_{total}) = k * u_c(\sigma_{total}) = 2 * 0.7191 = 1.4381 \ [N/mm^2]$$

This result is a percentage of the total tension that can be calculated as follows:
58.5 : 1.4381 = 100 : x

$$x = 2.458\ \%$$

Finally the global result for the calculation of the fatigue limit is:

$$58.55 \pm 2.458\%, \text{ with a coverage factor } k = 2$$

EXAMPLE 2 - Calculation of the relative standard uncertainty of the number of cycles N

The example is about the calculation of uncertainty of the number of cycles N.

Equation III.1 of ***Appendix III*** gives the uncertainty requested:

$$[u_c(N)/N]^2 = [-k * u(\sigma)/\sigma\ ]^2 + [k * u(\sigma_A)/\sigma_A\ ]^2 + [\ u(N_A)/N_A\ ]^2$$

DATA
k = 5.36 (from linear regression);

N_A = 2746008 [cycles] (number of cycles in point A. In this example fatigue limit has been chose as point A);

$$u(N_A) = 275\ (0.01\%\ \text{di}\ N_A\ k = 1)$$

σ_A = 58.5 [N/mm^2]
σ = stress bigger than σ_A = 100 [N/mm^2]
$S = \pi\ (D/2)^2 = 50.26548$ [mm^2]
$F_\sigma = \sigma * S = 100 * 50.26548 = 5026.55$ [N]
From the equation $N = N_A\ (\sigma / \sigma_A)^{-k}$: N = 155116 [cycles]

From **Example 1**:
$u(\sigma_A)$ = 0.7191 [N/mm^2], with a coverage factor k = 1

From table II - C and by Equation III.1:

$$u^2_c(\sigma) = [(1/50.27)^2 (26.09)^2 + (-5026.55 / 50.27^2)^2 (\pi\ 8.0/2)^2 (0.00879)^2]$$

$u(\sigma)$ = 0.56 [N/mm^2], with coverage factor k = 1

It is possible to calculate $u_c(N)/N$:

$$[u_c(N)/N]^2 = [-k * u(\sigma)/\sigma]^2 + [k * u(\sigma_A)/\sigma_A]^2 + [u(N_A)/N_A]^2$$

$$[u_c(N)/N]^2 = [-5.36 * 0.56/100]^2 + [5.36 * 0.7191/58.5]^2 + [275/2746008]^2$$

$$u_c(N)/N = 0.072$$

The uncertainty calculated corresponds to 68.5% probability of survival, that means a coverage factor k = 1.

We prefer to give the 95% probability of survival, with a coverage factor k = 2, so it is necessary to obtain the expanded standard uncertainty:

$$U(N)/N = k * u_c(N)/N = 2 * 0.072 = 0.144$$

Finally, the global result for the calculation of the fatigue limit is:

155116 [cycles] ± 14.4% (expanded standard uncertainty with a coverage factor k = 2).

Table I – A
Calculation - Uncertainty of the Diameter D [mm]

SOURCES OF UNCERTAINTY	**Influence**	**Type (A, B)**	**Divisor d_v**	**Value x_i / coverage factor k**	**Average X_m**	**Standard deviation** (*)	**Standard Uncertainty** (**) [mm]
TEST – PIECE							
Original section	n.a.						
Sensitivity of the Instrument		B	Rectangular √3	± 0.01	/	/	0.00577
Correct dimension of the diameter		A	/	8.01 8.03 7.99 8.00 8.01	8.008	0.014832	0.006633
COMBINED STANDARD UNCERTAINTY u_c(diameter) = $(\Sigma u^2_{s,i})^{1/2}$							**0.00879** [mm]

n.i. = not influential; n.a. = not applicable
(*) The standard deviation $s = (\Sigma^n_{i=1} (x_i-x_m)^2/(n-1))^{1/2}$
(**) The standard uncertainty is:
for an uncertainty type A : $u_{s,i} = s / (n^{1/2})$
for an uncertainty type B: $u_{s,i} = x * k'$

Table I – B
Calculation - Uncertainty of the Load F = 2764.6 [N]

SOURCES OF UNCERTAINTY	**Influence**	**Type (A, B)**	**Divisor d_v**	**Value x_i / coverage factor k**	**Average X_m**	**Standard deviation** (*)	**Standard Uncertainty** (**) [N]
APPARATUS							
Load Cell – calibration		B	Rectangular √3	0.5% (of the value F) k=1	/	/	7.9807
Load Cell – sensitivity		B	Rectangular √3	± 20 [N]	/	/	11.542
Dynamic control of load		B	Rectangular √3	0.5% (of the value F) k=1	/	/	7.9807
Drift of static control		B	Rectangular √3	0.1% (of the value F) k=1	/	/	2.7646
Tooling alignment		B	Rectangular √3	0.2% (of the value F) k=1	/	/	5.5292
Test frequency Type of waveform	n.i. n.i.						
COMBINED STANDARD UNCERTAINTY **u_c(load) = $(\Sigma u^2_{s,i})^{1/2}$**							**17.2896** [N]

n.i. = not influential; n.a. = not applicable; (*) and (**) = see notes to Table I - A

Table I – C
Calculation - Uncertainty of the Load F = 5027 [N]

SOURCES OF UNCERTAINTY	**Influence**	**Type (A, B)**	**Divisor d_v**	**Value x_i / coverage factor k**	**Average X_m**	**Standard deviation** (*)	**Standard Uncertainty** (**) [N]
APPARATUS							
Load Cell – calibration		B	Rectangular √3	0.5% (of the value F) k=1	/	/	14.512
Load Cell – sensitivity		B	Rectangular √3	± 20 [N]	/	/	11.542
Dynamic control of load		B	Rectangular √3	0.5% (of the value F) k=1	/	/	14.512
Drift of static control		B	Rectangular √3	0.1% (of the value F) k=1	/	/	5.027
Tooling alignment		B	Rectangular √3	0.2% (of the value F) k=1	/	/	10.054
Test frequency	n.i.						
Type of waveform	n.i.						
COMBINED STANDARD UNCERTAINTY **u_c(load) = $(\Sigma u^2_{s,i})^{1/2}$**							**26.09** [N]

n.i. = not influential; n.a. = not applicable, (*) and (**) = see notes to Table I - A

EXAMPLE 3

The example is about the calculation of the fatigue limit's uncertainty when the specimen is un-notched.

Equation I.1 of ***Appendix I,*** of the procedure Fatigue Test, gives the uncertainty requested:

$$u^2_c(\sigma_D) = [\ (1 / S)^2\ \mathbf{u^2_F\ (F_0)} + (-\ F_0 / S^2)^2\ (\pi\ D/2)^{\ 2}\ \mathbf{u^2(D)}] \qquad \text{I.1}$$

DATA
The specimen's nominal diameter is 7.50 mm;
alternative max. load F_0 is 4400 N;
Cross section S:
$S = \pi\ (D/2)^2 = 44\ [mm^2]$.
$\sigma_D = S_{A,\ kt=1} = 100\ [N/mm^2]$ this is the experimental result.
$k_t = 1$

$$u^2_c(\sigma_D) = [\ (1 / S)^2\ \mathbf{u^2_F\ (F_0)} + (-\ F_0 / S^2)^2\ (\pi\ D/2)^{\ 2}\ \mathbf{u^2(D)}] \qquad \text{I.2}$$

Table II - A is designed for simply obtaining the combined standard uncertainty of the *Diameter D*, $\mathbf{u_c\ (D)}$. All values can vary in accordance with each test method and apparatus; the user can use this table remembering to vary each term if necessary.

Table II - B is designed for simply obtaining the combined standard uncertainty of the *Load F*, $\mathbf{u_c(F_0)}$. All values can vary in accordance with each test method and apparatus; the user can use this table remembering to vary each term if necessary.

It has been obtained:

combined standard uncertainty $\mathbf{u_c(D)}$ = 0.00879 [mm];
combined standard uncertainty $\mathbf{u_c(F)}$ = 22.09 [N].

It is now possible to calculate the combined standard uncertainty of total stress σ_{total}:

$$u^2_c\ (\sigma_D) = [(1 /44)^2\ \mathbf{(22.09)^2} + (-\ 4400 / 44^2)^2\ (\ \pi\ 7.5/2)^2\ (\mathbf{0.00879)^2}]$$

$$u^2_c\ (\sigma_D) = 0.3074$$

$$u_c\ (\sigma_D) = u_c\ (S_{A,\ kt=1}\ ) = 0.5544\ [N/mm^2]$$

The uncertainty calculated corresponds to 68.5% probability of survival, that means a coverage factor k = 1.

The relative uncertainty is: $u_c(S_{A,\ Kt=1})/S_{A,\ Kt=1} = 0.5544/100 = 0.005544$.

We prefer to give the 95% probability of survival, with a coverage factor k = 2, so it is necessary to obtain the expanded standard uncertainty:

$U (S_{A, Kt=1}) = k * u_c(S_{A, Kt=1}) = 2 * 0.5544 = 1.108$ [N/mm^2]

This result is a percentage of the total tension that can be calculated as follows:
100: 1.108 = 100: x
x = 1.108 %

Finally, the global result for the calculation of the fatigue limit is:

100 MPa ± 1.015%, with a coverage factor k = 2

EXAMPLE 4

The example is about the calculation of the fatigue limit's uncertainty when the specimen is notched.

Equation II.1 of ***Appendix II,*** of the procedure Fatigue Test, gives the uncertainty requested:

$$u^2_c(\sigma_D) = [\ (1 / S)^2\, \mathbf{u^2_F\ (F_0)} + (- F_0 / S^2)^2 (\pi\ D/2)^2\, \mathbf{u^2(D)}] \qquad \text{II.1}$$

DATA
The specimen's nominal diameter is 12.00 mm;
the local diameter is 7.5 [mm];
alternative max. load F_0 is 2640 N;
Cross section S local:
$S = \pi\ (D/2)^2 = 44$ [mm^2].
$\sigma_D = S_{A, Kt>1} = 60$ [N/mm^2] this is the experimental result.
$K_t = 2.48$

$$u^2_c(\sigma_D) = [\ (1 / S)^2\, \mathbf{u^2_F\ (F_0)} + (- F_0 / S^2)^2 (\pi\ D/2)^2\, \mathbf{u^2(D)}] \qquad \text{II.2}$$

Table II - C is designed for simply obtaining the combined standard uncertainty of the Nominal diameter *D*, $\mathbf{u_c\ (D_n)}$. The all values can vary in accordance with each test method and apparatus; the user can use this table remembering to vary each term if necessary.

Table II - A is designed for simply obtaining the combined standard uncertainty of the local diameter *D*, $\mathbf{u_c\ (D_l)}$. The all values can vary in accordance with each test method and apparatus; the user can use this table remembering to vary each term if necessary.

Table II - D is designed for simply obtaining the combined standard uncertainty of the Load *F*, $\mathbf{u_c(F_0)}$. All values can vary in accordance with each test method and apparatus; the user can use this table remembering to vary each term if necessary.

It has been obtained:

combined standard uncertainty $\mathbf{u_c(D_n)}$ = 0.00879 [mm];
combined standard uncertainty $\mathbf{u_c(D_l)}$ = 0.00879 [mm];
combined standard uncertainty $\mathbf{u_c(F)}$ = 16.15 [N].

It is now possible to calculate the combined standard uncertainty of total stress σ_{total}:

$$u^2_c(\sigma_D) = [(1/44)^2 \mathbf{(16.15)^2} + (-2640/44^2)^2 (\pi\, 7.5/2)^2 (\mathbf{0.00879})^2]$$

$$u^2_c(\sigma_D) = 0.1546$$

$$u_c(\sigma_D) = u_c(S_{A, Kt>1}) = 0.3932 \text{ [N/mm}^2]$$

The uncertainty calculated corresponds to 68.5% probability of survival, that means a coverage factor k = 1.

The relative uncertainty is: $u_c(S_{A, Kt>1})/S_{A, Kt>1}$ = 0.3932/60 = 0.00655.

We prefer to give the 95% probability of survival, with a coverage factor k = 2, so it is necessary to obtain the expanded standard uncertainty:

$$U(S_{A, Kt=1}) = k * u_c(S_{A, Kt=1}) = 2 * 0.39 = 0.78 \text{ [N/mm}^2]$$

This result is a percentage of the total tension that can be calculated as follows:
60 : 0.78 = 100 : x
where x = 1.3 %

Finally, the global result for the calculation of the fatigue limit is:
60 ± 1.3%, with a coverage factor k = 2

EXAMPLE 5

This example is about the calculation of the combined standard uncertainty of the stress concentration factor K_t, $u_c(K_t)$.

In ***Appendix IV*** of this Procedure it is possible to find the right formula:

$$u^2_c(K_t) = (-a / D_l)^2\, u^2(r) + (a / D^2_l)^2\, u^2(D_l)$$

DATA
r = radius of the notch = 0.8 [mm];
D_l =diameter of the specimen at the notch = 7.5 [mm] ;
D_n = Nominal diameter = 12.0 [mm];
K_t = stress concentration factor

Calculation of the ratio r/D_l = 0.8/7.5 = 0.106;
Calculation of the ratio D_n/D_l = 12.0/7.5 = 1.6.

These values are entered in the (K_t - r/D_l) graphic whose output is the value $K_t = 2.48$. The graphic is a function also of the shape of the specimen and of the kind of load applied.

At the point (0.106; 2.48) of this graphic the user draws the tangent to the curve having ratio $D_n/D_l = 1.6$.

Now it is possible to obtain the equation of this tangent. In our case the equation is:
$K_t = - 10.625\ (r/D_l) + 3.64$
a = 10.625

From Table II - A:
combined standard uncertainty $\mathbf{u_c(D_l)}$ = 0.00879 [mm];
From Table II - E:
combined standard uncertainty $\mathbf{u_c(r)}$ = 0.00879 [mm];

Using the general expression written above it is possible to obtain the uncertainty of the stress concentration factor:

$$u^2_c(K_t) = (-a / D_l)^2\, u^2(r) + (a / D^2_l)^2\, u^2(D_l)$$

$$u^2_c(K_t) = (-10.625 / 7.5)^2\, (0.00879)^2 + (10.625 / 7.5^2)^2\, (0.00879)^2$$

$$u^2_c(K_t) = 0.0001578$$

$$u_c(K_t) = 0.0126$$

The uncertainty calculated corresponds to 68.5% probability of survival, that means a coverage factor k = 1.

We prefer to give the 95% probability of survival, with a coverage factor k = 2, so it is necessary to obtain the expanded standard uncertainty:

$$U(K_t) = k * u_c(K_t) = 2 * 0.0126 = 0.025$$

This result is a percentage of the stress concentration factor that can be calculated as follows:

2.48: 0.025 = 100: x   x = 1.013 %

the global result for the calculation of the stress concentration factor is:

2.48 ± 1.013%, with a coverage factor k = 2

It is also possible to calculate the relative combined standard uncertainty:

$u_c(K_t)/ K_t = 0.0126 / 2.48 = 0.00508$ with a coverage factor k = 1.
$u_c(K_t)/ K_t = 0.0126 * 2 / 2.48 = 0.01016$ with a coverage factor k = 2.

EXAMPLE 6

This example is about the calculation of the uncertainty of the fatigue notch factor K_f.

In ***Appendix V*** of this Procedure it is possible to find the formula for the relative uncertainty:

$$[u_c(K_f)/K_f]^2 = [1 * u(S_{A,\,Kt=1})/\ S_{A,\,Kt=1}]^2 + [-1 * u(S_{A,\,Kt>1})/\ S_{A,\,Kt>1}]^2$$

DATA from Examples 3, 4, 5:

K_f = fatigue notch factor = 1.66;
$u_c(S_{A,\,Kt=1})$= uncertainty of the fatigue limit of an un-notched specimen = 0.5544, k =1;
$u_c(S_{A,\,Kt>1})$ = uncertainty of the fatigue limit of a notched specimen = 0.3932, k =1;
$S_{A,\,Kt=1}$ = 100 [N/mm^2];
$S_{A,\,Kt>1}$ = 60 [N/mm^2].

The user can calculate the fatigue notch factor $K_f = S_{A,\,Kt=1} / S_{A,\,Kt>1}$ = 100/60 = 1.66
and it's relative uncertainty:

$$[u_c(K_f)/K_f]^2 = [1 * 0.5544/100]^2 + [-1 * 0.3932/60]^2$$

$$[u_c(K_f)/K_f]^2 = 0.000073682$$

$$[u_c(K_f)/K_f] = 0.008584$$

The combined standard uncertainty is:

$$[u_c(K_f)] = [u_c(K_f)/K_f] * K_f = 0.008584 * 1.66 = 0.0142$$

The uncertainty calculated corresponds to 68.5% probability of survival, that means a coverage factor k = 1.
We prefer to give the 95% probability of survival, with a coverage factor k = 2, so it is necessary to obtain the expanded standard uncertainty:

$$U(K_f) = k * u_c\ (K_f) = 2 * 0.0142 = 0.0285$$

This result is a percentage of the fatigue notch factor that can be calculated as follows:

1.66 : 0.0285 = 100 : x　　　　　　x = 1.717 %

the global result for the calculation of the fatigue notch factor is:

1.66 ± 1.717 %, with a coverage factor k = 2

EXAMPLE 7

This example is about the calculation of the uncertainty of the notch sensitivity Q.

In ***Appendix VI*** of this Procedure it is possible to find the formula for the uncertainty:

$u^2_c(Q) = [1/ (K_t - 1)]^2 u^2(K_f) + [(-K_f + 1) / (K_t - 1)^2]^2 u^2(K_t)$

DATA from Examples 3, 4, 5, 6:

K_f = fatigue notch factor =1.66;
K_t = stress concentration factor = 2.48;
Q = notch sensitivity;
the combined standard uncertainty of K_f is: $[u_c(K_f)] = 0.0142$
the combined standard uncertainty of K_t is: $[u_c(K_t)] = 0.0126$

The formula for the calculates of the notch sensitivity is:

$$Q = (K_f - 1) / (K_t - 1) = (1.66 - 1)/(2.48 - 1) = 0.446$$

Calculation of the standard uncertainty:

$$u^2_c(Q) = [1/1.48]^2 \ 0.0142^2 + [-0.66 / 1.48^2]^{2} \ 0.0126^2$$

$$u^2_c(Q) = 0.00010697$$

$$u_c(Q) = 0.01034$$

The uncertainty calculated corresponds to 68.5% probability of survival, that means a coverage factor k = 1.

We prefer to give the 95% probability of survival, with a coverage factor k = 2, so it is necessary to obtain the expanded standard uncertainty:

$$U(Q) = k * u_c (Q) = 2 * 0.01034 = 0.02068$$

This result is a percentage of the notch sensitivity that can be calculated as follows:

0.446: 0.02068 = 100: x   x = 4.64 %

the global result for the calculation of the notch sensitivity is:

0.446 ± 4.64 %, with a coverage factor k = 2

Table II – A
Calculation - Uncertainty Diameter D = 7.5 [mm]
Diameters: D_n for un-notched specimen D_l for notched specimen

SOURCES OF UNCERTAINTY	**Influence**	**Type (A, B)**	**Divisor d_v**	**Value x_i / coverage factor k**	**Average x_m**	**Standard deviation** (*)	**Standard Uncertainty** (**) [mm]
TEST – PIECE							
Original section	n.a.						
Sensitivity of the Instrument		B	Rectangular √3	± 0.01	/	/	0.00577
Correct dimension of the diameter		A	/	7.51 7.53 7.49 7.50 7.51	7.508	0.014832	0.006633
COMBINED STANDARD UNCERTAINTY **u_c(diameter) = $(\Sigma u^2_{s,i})^{1/2}$**							**0.00879** [mm]

n.i. = not influential; n.a. = not applicable
(*) The standard deviation $s = (\Sigma^n_{i=1} (x_i-x_m)^2/(n-1))^{1/2}$
(**) The standard uncertainty is:
for an uncertainty type A : $u_{s,i} = s / (n^{1/2})$
for an uncertainty type B: $u_{s,i} = x * k'$

Table II – B
Calculation - Uncertainty of the Load F = 4400 [N]
Load for Un-notched Specimen

SOURCES OF UNCERTAINTY	**Influence**	**Type (A, B)**	**Divisor d_v**	**Value x_i / coverage Factor k**	**Average x_m**	**Standard deviation** (*)	**Standard Uncertainty** (**) [N]
APPARATUS							
Load Cell – calibration		B	Rectangular √3	0.5% (of the value F) k=1	/	/	12.7
Load Cell – sensitivity		B	Rectangular √3	± 20 [N]	/	/	11.542
Dynamic control of load		B	Rectangular √3	0.5% (of the value F) k=1	/	/	12.7
Drift of static control		B	Rectangular √3	0.1% (of the value F) k=1	/	/	2.57
Machine alignment		B	Rectangular √3	0.2% (of the value F) k=1	/	/	5.08
Test frequency Type of waveform	n.i. n.i.						
COMBINED STANDARD UNCERTAINTY u_c(load) = ($\Sigma u^2_{s,i}$)$^{1/2}$							**22.09** [N]

n.i. = not influential; n.a. = not applicable
(*), (**) see notes of Table II - A

Table II - C
Calculation - Uncertainty of the Diameter D= 12 [mm]
Diameter: D_n for notched Specimen

SOURCES OF UNCERTAINTY	**Influence**	**Type (A, B)**	**Divisor d_v**	**Value x_i / coverage Factor k**	**Average X_m**	**Standard deviation** (*)	**Standard Uncertainty** (**) [mm]
TEST – PIECE							
Original section	n.a.						
Sensitivity of the		B	Rectangular	± 0.01	/	/	0.00577
Instrument			√3				
Correct dimension of the		A	/	12.01	12.008	0.014832	0.006633
Diameter				12.03			
				11.99			
				12.00 12.01			
COMBINED STANDARD UNCERTAINTY u_c(diameter) = ($\Sigma u^2_{s,i}$)$^{1/2}$							**0.00879** [mm]

n.i. = not influential; n.a. = not applicable
(*), (**) see notes of Table II - A

Table II - D
Calculation - Uncertainty of the Load F = 2640 [N]
Load for Notched Specimen

SOURCES OF UNCERTAINTY	**Influence**	**Type (A, B)**	**Divisor d_v**	**Value x_i / coverage Factor k**	**Average x_m**	**Standard deviation** (*)	**Standard uncertainty** (**) [N]
APPARATUS							
Load Cell - calibration		B	Rectangular √3	0.5% (of the value F) k=1	/	/	7.62
Load Cell - sensitivity		B	Rectangular √3	± 20 [N]	/	/	11.542
Dynamic control of load		B	Rectangular √3	0.5% (of the value F) k=1	/	/	7.62
Drift of static control		B	Rectangular √3	0.1% (of the value F) k=1	/	/	1.524
Machine alignment		B	Rectangular √3	0.2% (of the value F) k=1	/	/	3.048
Test frequency	n.i.						
Type of waveform	n.i.						
COMBINED STANDARD UNCERTAINTY $u_c(load) = (\Sigma u^2_{s,i})^{1/2}$							**16.15** [N]

n.i. = not influential; n.a. = not applicable
(*), (**) see notes of Table II - A

Table II - E
Calculation - Uncertainty Radius r = 0.8 [mm]
Radius of the notched Specimen

SOURCES OF UNCERTAINTY	**Influence**	**Type (A, B)**	**Distribution / factor k'**	**Value x_i / coverage Factor k**	**Average x_m**	**Standard deviation** (*)	**Standard uncertainty** (**) [mm]
TEST – PIECE							
Original section	n.a.						
Sensitivity of the Instrument		B	Rectangular 1/√3	± 0.01	/	/	0.00577
Correct dimension of the Diameter		A	/	0.81 0.83 0.79 0.8 0.81	0.808	0.014832	0.006633
COMBINED STANDARD UNCERTAINTY $u_c(radius) = (\Sigma u^2_{s,i})^{1/2}$							**0.00879** [mm]

n.i. = not influential; n.a. = not applicable
(*), (**) see notes of Table II - A