Manual of Codes of Practice for the Determination of Uncertainties in Mechanical Tests on Metallic Materials

Code of Practice No. 02

The Determination of Uncertainties in Low Cycle Fatigue Testing

F A Kandil

National Physical Laboratory
Queens Road
Teddington, Middlesex TW11 0LW
UNITED KINGDOM

Issue 1

September 2000

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1. SCOPE

This procedure covers the evaluation of uncertainty in strain-controlled low-cycle fatigue (LCF) test results obtained from tests at ambient or elevated temperature and carried out according to any of the following standards and draft standard:

ISO/DIS 12106, "*Metallic Materials - Fatigue Testing - Axial Strain-Controlled Method*", August 1998.

ASTM E606-92, "*Standard Practice for Strain-Controlled Fatigue Testing*", 1998 Annual Book of ASTM Standards, Section 3, Volume 03.01.

PrEN 3988:1998,"*Aerospace Series - Test Methods for Metallic Materials - Constant Amplitude Strain-Controlled Low Cycle Fatigue Testing*", AECMA, Draft No.2, March 1998.

BS 7270:1990, "*Method for Constant Amplitude Strain Controlled Fatigue Testing*", British Standards Institution, 1990.

The procedure is restricted to uniaxial LCF tests conducted in strain control at constant-amplitude, and for temperature and strain rate combinations that produce no time-dependent behaviour. The tests are assumed to run continuously without interruption on specimens with uniform gauge lengths.

2. SYMBOLS AND DEFINITIONS

For a complete list of uncertainty symbols and definitions, see Section 2 of the main *Manual* [1]. It should be noted that not all the symbols and definitions used in this Code of Practice are consistent with the GUM [2]. In a few cases there are conflicts between the symbols used in the above mentioned test standards and the GUM. In such cases the test Standards are given preference.

The following list gives the symbols and definitions used in this procedure. It should be noted that the definition of the plastic strain range component, $\Delta\varepsilon_p$, used here is consistent with both ISO/DIS 12106 and BS 7270:1990 but differs slightly from the definitions adopted in the ASTM E606-92 and PrEN 3988:1998 procedures (see Ref. [3]).

A_o	specimen's original cross-sectional area
c_i	sensitivity coefficient
c_T	sensitivity coefficient that describes the variation of fatigue life at a given total strain range as a function of the test temperature [see Eq. (A35)]
COD	coefficient of determination
CoP	code of Practice
d	specimen diameter
d_v	divisor used to calculate the standard uncertainty

e	extension
E_o	Young's modulus of elasticity determined from the initial loading of the first cycle or prior to the start of the test
E_1, E_2	values of the tangent modulus of elasticity determined on the unloading and loading segments respectively, of the stress-strain hysteresis loop nearest to mid-life (see Fig.1)
F	applied force
k	coverage factor used to calculate the expanded uncertainty where a normal probability distribution can be assumed. The expanded uncertainty usually corresponds to the 95% confidence level.
k_p	coverage factor used to calculate an expanded uncertainty where a normal probability distribution cannot be assumed (see the *Manual* [1], Section 2). The expanded uncertainty usually corresponds to the 95% confidence level.
l_g	gauge length
m	number of input quantities on which the measurand depends
n	number of repeat measurements
N	number of (strain) cycles in a fatigue test
N_f	number of cycles to failure (In the example given in Appendix B, this is defined as the number of cycles to failure corresponding to a 25% drop in maximum tensile stress.)
p	confidence level
q	random variable
$\bar{q}$	arithmetic mean of q
s	experimental standard deviation (of a random variable) determined from a limited number of measurements, n
t	original thickness of a rectangular specimen
T	nominal test temperature (in degrees Celsius or Kelvin, as indicated)
u_i	standard uncertainty
u_c	combined standard uncertainty
$u(N_f)_{det}$	estimated uncertainty due to the method of determining N_f
$u(N_f)_{rep}$	estimated uncertainty in the mean value of N_f in a series of identical tests (i.e. repeatability of the measurement)
U	expanded uncertainty
V	value of the measurand
w	original width of a rectangular specimen
x_i	estimate of input quantity
y	test (or measurement) result
α	slope of the tangent to the log $\Delta\varepsilon_t$ vs log N_f curve [see Fig. (A1)]
β	percent bending (see Ref. [4])
δ_T	combined estimated error in the temperature measurement and control
δ_{Tc}	error in temperature control (i.e. the difference between the indicated nominal temperature and the test nominal temperature)
δ_{Ts}	error in temperature stability (i.e. variability in the indicated reading of a given thermocouple during the fatigue test)
δ_{Tt}	error in the thermocouple indicated temperature (estimated from thermocouple calibration)

d_{Tu}	error in the temperature uniformity along the gauge length (i.e. temperature excursions about the indicated nominal temperature)
d_e	error in extension measurement
d_{gl}	error in the extensometer gauge length (due to resetting of the indicated strain reading at the beginning of each test)
d_{De}	estimated error in the measurement of the total strain range
e_{bk}	maximum bending strain at the maximum peak strain in the fatigue cycle (see Ref. [4])
e_{bv}	maximum bending strain at the minimum peak strain in the fatigue cycle (see Ref. [4])
De_o	average axial strain range measured by the strain gauges (in specimen bending measurement)
De_b	bending strain range (= e_{bk} - e_{bv})
De_t	total strain range
De_p	plastic strain range (the width of the hysteresis loop, determined at the mean stress)
Ds	stress range (= s_{max} - s_{min})
s	stress
s_m	mean stress (= ½[s_{max} + s_{min}])
s_{max}	maximum stress
s_{min}	minimum stress
n_i	degrees of freedom of standard uncertainty u_i (see the *Manual* [1], Section 2)
n_{eff}	effective degrees of freedom used to obtain k_p (see *Manual* [1], Section 2)
Y	Bending Reversibility Parameter (= abs (De_b / De_o), see Ref. [4])

Figure 1 shows some of the definitions of the parameters used in the fatigue test, where De_p is in accordance with ISO/DIS 12106.

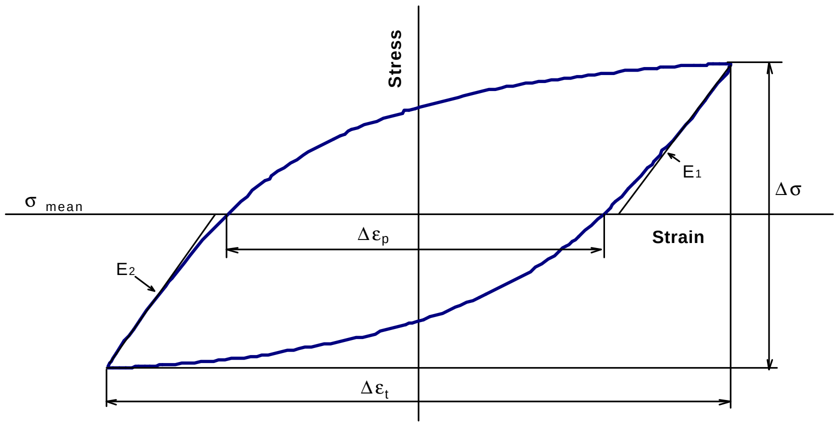

Fig.1 Parameters used in strain-controlled fatigue testing.

3. INTRODUCTION

There are requirements for test laboratories to evaluate and report the uncertainty associated with their test results. Such requirements may be demanded by a customer who wishes to know the limits within which the reported result may be reasonably assumed to lie; or the laboratory itself may want to gain a better understanding of which aspects of the test procedure have the greatest effect on results so that this may be monitored more closely or improved. This Code of Practice (CoP) has been prepared within UNCERT, a project partially funded by the European Commission's Standards, Measurement and Testing programme under reference SMT4 -CT97-2165, to simplify the way in which uncertainties in mechanical tests on metallic materials are evaluated. The aim is to avoid ambiguity and provide a common format readily understandable by customers, test laboratories and accreditation authorities.

This CoP is one of seventeen produced by the UNCERT consortium for the estimation of uncertainties associated with mechanical tests on metallic materials. The Codes of Practice have been collated in a single *Manual* [1] that has the following sections:

1. Introduction to the evaluation of uncertainty
2. Glossary of definitions and symbols
3. Typical sources of uncertainty in materials testing
4. Guidelines for the estimation of uncertainty for a series of tests
5. Guidelines for reporting uncertainty
6. Individual Codes of Practice (of which this is one) for estimating the uncertainties in specific mechanical tests on metallic materials

This CoP can be used as a stand-alone document. For further background information on measurement uncertainty and values of standard uncertainties of the equipment and instrumentation used commonly in material testing, the user may need to refer to Section 3 of the *Manual* [1]. The individual CoPs are kept as simple as possible by following the same structure; viz:

- The main procedure.
- Details of the uncertainty calculations for the particular test type (Appendix A).
- A worked example (Appendix B).

This CoP guides the user through the various steps to be carried out to estimate the uncertainty in LCF results obtained from a single test or a series of tests. An introduction describing the general process for calculating uncertainty values is given in Section 1 of the *Manual* [1].

4. A PROCEDURE FOR ESTIMATING THE UNCERTAINTIES IN LOW CYCLE FATIGUE TESTING

Step 1. Identifying the Parameters for which Uncertainty is to be Estimated

The first step is to list the quantities (measurands) for which the uncertainties must be calculated. Table 1 shows the parameters that are usually reported in strain-controlled low-cycle fatigue testing. None of these measurands are measured directly, but are determined from other quantities (or measurements).

Table 1 Measurands, measurements, their units and symbols.

Measurands	Units	Symbol
Fatigue life	cycles (dimensionless)	N_f
Modulus of Elasticity 1)	GPa	E_o, E_1, E_2
Maximum stress 2)	MPa	s_{max}
Minimum stress 2)	MPa	s_{min}
Plastic strain range 2)	dimensionless	De_p

Measurements	Units	Symbol
Specimen diameter	mm	d
Specimen bending	dimensionless	Y
Gauge length	mm	l_g
Force	kN	F
Extension	mm	e
Temperature	^{o}C	T
Number of cycles	cycles	N

1) initial value (E_o) and at mid-life (E_1 and E_2).
2) at mid-life.

Step 2. Identifying all Sources of Uncertainty in the Test

In Step 2, the user must identify all possible sources of uncertainty that may have an effect (either directly or indirectly) on the test. The list cannot be identified comprehensively beforehand, as it is associated uniquely with the individual test procedure and apparatus used. This means that a new list should be prepared each time a particular test parameter changes (e.g. when a plotter is replaced by a computer). To help the user list all sources, four categories have been defined for this particular test arrangement, and Table 2 lists these categories and gives some examples of sources of uncertainty in each category. It should be noted that these measurement uncertainties do not include those due to human error.

It is important to note that Table 2 is **not** exhaustive and is for **guidance** only - relative contributions may vary according to the material tested and the test conditions. Individual laboratories are encouraged to prepare their own list to correspond to their particular test facility and assess the associated significance of the contributions.

Table 2 Typical sources of uncertainty and their likely contribution to the uncertainties of strain-controlled LCF measurands for a typical superalloy at elevated temperature.

[1 = major contribution, 2 = minor contribution, blank = insignificant (or no) contribution, ? = unknown]

Source of uncertainty	**Type** [1)]	**Measurand**			
		N_f	E_o, E_1, E_2	s_{max}, s_{min}	De_p
1. Test piece					
Diameter	*B*	2	1	1	2
Bending	*B*	1	2	2	2
Surface finish	*B*	2			
Residual stresses	*B*	?	?	?	?
2. Test system					
Alignment[2)]	*B*	1	2	2	2
Uncertainty in force measurement	*B*		1	1	2
Drift in force measuring system	*B*	2		1	2
Uncertainty in strain measurement	*B*	1	1	1	2
Drift in strain measuring system	*B*	2	2	2	2
Gauge length (due to resetting zero reading)	*B*	1	1		2
Uncertainty in controlling strain limits	*B*	2		2	2
Thermocouple indicated reading	*B*	1	2	2	2
Thermocouple drift	*B*	2	2	2	2
Non-uniformity of specimen temperature	*B*	1	2	2	2
Temperature fluctuations	*B*	1	2	1	2
3. Environment					
Laboratory ambient temperature and humidity	*B*	2	2	2	2
4. Test Procedure					
Soaking time[3)]	*B*	2	2	2	2
Strain rate (or cycle frequency)	*B*	2	2	2	2
Method of determining N_f	*B*	1		2	2
Repeatability of N_f	*A*	1			

1) for definitions, see Step 3.
2) affects specimen bending.
3) assuming adherence to a set time, typically 1 hour ± 15 minutes.

To simplify the calculations it is advisable to group the significant sources of uncertainty in Table 2, into the following categories:

1 Uncertainty in fatigue life due to specimen bending (which results from misalignment in the test system and/or dimensional non-compliance in the test piece).

2 Uncertainty in the strain measurement and control (which combines extensometer calibration and errors in the gauge length due to extensometer resetting).

3 Uncertainty in the temperature measurement and control (which combines errors in the indicated reading, non-uniformity of specimen temperature within the gauge length, and temperature fluctuations during the test).

4 Uncertainty in fatigue life due to the method of determining $N_{f.}$ (which depends on whether N_f is determined manually from graphs or using computer).

5 Uncertainty in the mean value of N_f (i.e. repeatability of the measurement).

6 Uncertainties in the stress values.

7 Uncertainty in the Young's modulus of elasticity.

8 Uncertainty in the plastic strain range component.

Appendix A and the worked example in Appendix B use the above categorisation when assessing uncertainties.

Step 3. Classifying the Uncertainty According to *Type A* or *Type B*

In this third step, which is in accordance with the GUM [2], the sources of uncertainty are classified as *Type A* or *B*, depending on the way their influence is quantified. If the uncertainty is evaluated by statistical means (from a number of repeated observations), it is classified as *Type A*. If it is evaluated by any other means it should be classified as *Type B*.

The values associated with *Type B* uncertainties can be obtained from a number of sources including calibration certificates, manufacturer's information, or an expert's estimation. For *Type B* uncertainties, it is necessary for the user to estimate for each source the most appropriate probability distribution (further details are given in Section 2 of the *Manual* [1]).

Step 4. Estimating the Standard Uncertainty and Sensitivity Coefficient for each Source of Uncertainty

In this step the standard uncertainty, u, for each major input source identified in Table 2 is estimated (see Appendix A). The standard uncertainty is defined as one standard deviation and is derived from the uncertainty of the input quantity divided by the parameter, d_v, associated with the assumed probability distribution. The divisors for the typical distributions most likely to be encountered are given in Section 2 of the *Manual* [1].

The standard uncertainty requires the determination of the associated sensitivity coefficient, c_i, which is usually estimated from the partial derivatives of the functional relationship between the output quantity (the measurand) and the input quantities. The calculations required to obtain the sensitivity coefficients by partial differentiation can be a lengthy process, particularly when there are many individual contributions and uncertainty estimates are needed for a range of values. If the functional relationship for a particular measurement is not known, the sensitivity coefficients may be obtained experimentally. In many cases the input quantity may not be in the same units as the output quantity. For example, one contribution to N_f is the test temperature. In this case the input quantity is temperature, but the output quantity is the number of cycles to failure which is dimensionless. In such cases, a sensitivity coefficient, c_T (corresponding to the partial derivative of the N_f / test temperature relationship), is used to convert from temperature to the number of cycles to failure (see example in Fig. A3).

To help with the calculations, it is useful to summarise the uncertainty analysis in a spreadsheet - or ‘uncertainty budget’- as in Table 3 below. Appendix A includes the mathematical formulae for calculating the uncertainty contributions and Appendix B gives a worked example.

Table 3 A Typical Uncertainty Budget Worksheet for Calculating the Uncertainty in LCF Life in a Series of Strain-Controlled Tests at Elevated Temperature.

Source of uncertainty	Symbol	Value	Probability distribution	Divisor d_v	c_i	$u_i(N_f)$ cycles	n_i or n_{eff}
Specimen bending [1)]	y		Rectangular	√3	$1/a$		∞
Strain measurement [2)]	De		Rectangular	√3	$1/a$		∞
Temperature measurement [3)]	T		Rectangular	√3	c_T		∞
Method of determining N_f [4)]	$u(N_f)_{det}$		Rectangular	√3	1.0		∞
Repeatability of N_f [5)]	$u(N_f)_{rep}$		Normal	1.0	1.0		n-1
Combined standard uncertainty [6)]	u_c		Normal			$u_c(N_f)$	n_{eff}
Expanded uncertainty [7)]	U		Normal				n_{eff}

1) See Section A5.
2) Includes all contributions due to errors in strain measurement and control (Section A6.)
3) Includes all contributions due to errors in specimen temperature measurement and control (Section A7.)
4) See Section A8.
5) For single-test calculations, this source is obviously not relevant (Section A9.)
6) See Step 5.
7) See Step 6.

Step 5. Computing the Combined Uncertainty u_c

Assuming that individual uncertainty sources are uncorrelated, the combined uncertainty of the measurand, $u_c(y)$, can be computed using the root sum squares:

$$u_c(y) = \sqrt{\sum_{i=1}^{m} [c_i u(x_i)]^2} \tag{1}$$

where c_i is the sensitivity coefficient associated with the input quantity x_i. The combined uncertainty corresponds to plus or minus one standard deviation and, therefore, has an associated confidence level of 68.27%.

Step 6. Computing The Expanded Uncertainty *U*

The expanded uncertainty U is defined in the GUM [2] as "the interval about the result of a measurement that may be expected to encompass a large fraction of the distribution of values that could **reasonably** be attributed to the measurand". It is obtained by multiplying the combined uncertainty u_c calculated in Step 5, by a coverage factor, k or k_p, which is selected on the basis of the level of confidence required. For a normal probability distribution a coverage factor of 2 is most commonly used and this corresponds to a confidence interval of 95.4% (effectively 95% for most practical purposes). The expanded uncertainty U is, therefore, broader than the combined uncertainty, u_c. Where a higher confidence level is demanded by the customer, (such as for particular measurements in the aerospace and electronics industries), a coverage factor of 3 or more is sometimes used. For a coverage factor of 3, the corresponding confidence level is 99.73%.

In cases where the probability distribution of u_c is not normal or where the number of data points used in a *Type A* analysis is small, a coverage factor k_p should be determined according the degrees of freedom given by the Welsh-Satterthwaite method (see Section 4 of the *Manual* [1] for more details).

Step 7. Reporting of Results

Once the expanded uncertainty has been estimated, the results should be reported in the following way:

$$V = y \pm U \tag{2}$$

where V is the estimated value of the measurand, y is the test (or measurement) result, U is the expanded uncertainty associated with y. An explanatory note, such as that given in the following example should be added (change as appropriate):

The reported expanded uncertainty is based on a standard uncertainty multiplied by a coverage factor, k = 2, which provides a level of confidence of approximately 95%. The uncertainty evaluation was carried out in accordance with UNCERT CoP 02: 2000.

5. REFERENCES

1. *Manual of Codes of Practice for the determination of uncertainties in mechanical tests on metallic materials*. Project UNCERT, EU Contract SMT4-CT97-2165, Standards Measurement & Testing Programme, ISBN 0-946754-41-1, Issue 1, September 2000.

2. BIPM, IEC, IFCC, ISO, IUPAC, OIML, "*Guide to the expression of uncertainty in measurement*". International Organisation for Standardisation, Geneva, Switzerland, ISBN 92-67-10188-9, First Edition, 1993. (This *Guide* is often referred to as the GUM or the ISO TAG4 document after the ISO Technical Advisory Group that produced it.)

   Identical documents:
   - ENV 13005:1999 (English)
   - NF ENV 13005:1999 (French)
   - NEN NVN ENV 13005:1999 (Dutch)
   - "*Vocabulary of metrology, Part 3. Guide to the expression of uncertainty in measurement*", PD 6461: Part 3: 1995, British Standards Institution.

3. F A Kandil, *"Potential ambiguity in the determination of the plastic strain range component in LCF testing"*, Int. J. Fatigue 21 (1999), 1013-18.

4. F A Kandil, "*Code of Practice for the measurement of bending in uniaxial low cycle fatigue testing*". Best Practice in Measurement Series, NPL MMS 001:1995, ISBN 0-946754-16-0, National Physical Laboratory, UK. Reprinted as Measurement Good Practice Guide No. 1, NPL, ISSN 1368-6550, March 1998.

5. F A Kandil, "*A procedure for quantifying uncertainty in LCF testing*", NPL Report CMMT (D) 163, National Physical Laboratory, UK, October 1998.

ACKNOWLEDGEMENTS

This document was written as part of project UNCERT, "*Code of Practice for the Determination of Uncertainties in Mechanical Tests on Metallic Materials*". The project was partly funded by the Commission of European Communities through the Standards, Measurement and Testing Programme, Contract No. SMT4-CT97-2165. Financial support was received by NPL from the United Kingdom Department of Trade and Industry as part of the Materials Measurement programme of underpinning research.

The author gratefully acknowledges the helpful comments made by Mr P McCarthy of ERA Technology.

APPENDIX A

MATHEMATICAL FORMULAE FOR CALCULATING UNCERTAINTIES IN LCF TEST RESULTS AT ELEVATED TEMPERATURES

A1. Uncertainty due to Errors in Determining the Cross-Sectional Area

A1.1 For a Circular Cross-Section

$$A_o = \frac{\pi d^2}{4} \qquad \text{(A1)}$$

The sensitivity coefficient c_i associated with the uncertainty in d is:

$$\frac{\partial A_o}{\partial d} = \frac{\pi d}{2} \qquad \text{(A2)}$$

and the uncertainty in A_o is:

$$u_{Ao} = \sqrt{\frac{\pi^2 d^2 u_d^2}{4}} \qquad \text{(A3)}$$

This can be expressed in relative terms:

$$\frac{u_{Ao}}{A_o} = 2\frac{u_d}{d} \qquad \text{(A4)}$$

A1.2 For a Rectangular Cross-Section

$$A_o = tw \qquad \text{(A5)}$$

The sensitivity coefficients c_i associated with the uncertainties in t and w are:

$$\frac{\partial A_o}{\partial t} = w \qquad \text{(A6)}$$

and $$\frac{\partial A_o}{\partial w} = t \qquad \text{(A7)}$$

and the uncertainty in A_o is:

$$u_{A_o} = \sqrt{w^2 u_t^2 + t^2 u_w^2} \qquad \text{(A8)}$$

Note that Eq. (A8) is based on the assumption that there is no correlation between t and w. However, in practice, the same measuring instrument (a micrometer or a vernier calliper) is usually used for measuring both t and w and, therefore, they are correlated. In this case it is suggested to add the standard uncertainties arithmetically, thus:

$$u_{Ao} = u_t + u_w \tag{A9}$$

The GUM [2] should be consulted for a detailed approach on the treatment of correlated contributions.

A2. Uncertainty in Stress

$$\sigma = \frac{F}{A_0} \tag{A10}$$

The sensitivity coefficients c_i associated with the uncertainty in F and A_o are:

$$\frac{\partial \sigma}{\partial F} = \frac{1}{A_o} \tag{A11}$$

$$\frac{\partial \sigma}{\partial A_o} = -\frac{F}{A_o^2} \tag{A12}$$

and the uncertainty in σ is:

$$u_\sigma = \sqrt{\left(\frac{1}{A_o}\right)^2 u_F^2 + \left(\frac{F}{A_o^2}\right)^2 u_{A_o}^2} \tag{A13}$$

Equation (A13) can be expressed in relative uncertainty as:

$$\frac{u_\sigma}{\sigma} = \sqrt{\left(\frac{u_F}{F}\right)^2 + \left(\frac{u_{Ao}}{A_o}\right)^2} \tag{A14}$$

Note that in Eqs. (A13) and (A14), it is assumed that both u_F and u_{Ao} have a normal probability distribution. For rectangular distributions divide these terms by $\sqrt{3}$.

A3. Uncertainty in Strain

The displacement *e* and the strain *e* are expressed as:

$$e = \delta L \tag{A15}$$

$$\varepsilon = \frac{e}{L_o} \tag{A16}$$

The sensitivity coefficients c_i associated with the uncertainties in *e* and L_o are:

$$\frac{\partial \varepsilon}{\partial e} = \frac{1}{L_o} \tag{A17}$$

$$\frac{\partial \varepsilon}{\partial L_o} = -\frac{e}{L_o^2} \tag{A18}$$

and the uncertainty in u_e is:

$$u_\varepsilon = \sqrt{\left(\frac{1}{L_o}\right)^2 u_e^2 + \left(\frac{e}{L_o^2}\right)^2 u_{Lo}^2} \tag{A19}$$

Equation (A19) can be expressed in relative uncertainty as:

$$\frac{u_\varepsilon}{\varepsilon} = \sqrt{\left(\frac{u_e}{e}\right)^2 + \left(\frac{u_{Lo}}{L_o}\right)^2} \tag{A20}$$

Note that in Eqs. (A19) and (A20), it is assumed that both u_e and u_{Lo} have a normal probability distribution. For rectangular distributions divide these terms by $\sqrt{3}$.

A4. Uncertainty in Young's Modulus of Elasticity

The Young's modulus measurement should be the last step in the preparation for the fatigue test to provide a good indication of whether the test set-up has been performed correctly. In this case the main sources of the *Type B* components of uncertainty are associated with the measurements of stress [Eq. (A13) or (A14)] and strain [Eq. (A19) or (A20)]. The standard uncertainty in *E* may be derived as follows:

$$E = \frac{\sigma}{\varepsilon} \tag{A21}$$

and the sensitivity coefficients c_i associated with the uncertainties in σ and ε are:

$$\frac{\partial E}{\partial \sigma} = \frac{1}{\varepsilon} \tag{A22}$$

$$\frac{\partial E}{\partial \varepsilon} = -\frac{\sigma}{\varepsilon^2} \tag{A23}$$

The uncertainty u_E is:

$$u_E = \sqrt{\left(\frac{1}{\varepsilon}\right)^2 u_\sigma^2 + \left(\frac{\sigma}{\varepsilon^2}\right)^2 u_\varepsilon^2} \tag{A24}$$

Or

$$\frac{u_E}{E} = \sqrt{\left(\frac{u_\sigma}{\sigma}\right)^2 + \left(\frac{u_\varepsilon}{\varepsilon}\right)^2} \tag{A25}$$

Where $\frac{u_E}{E}$, $\frac{u_\sigma}{\sigma}$ and $\frac{u_\varepsilon}{\varepsilon}$ are the relative standard uncertainties in Young's modulus, the applied stress and the strain, respectively. Note that the uncertainty in strain includes the error in measuring the extension and the error in the gage length due to resetting the extensometer.

In Eqs. (A24) and (A25), it is assumed that both u_σ and u_ε have a normal probability distribution. For rectangular distributions divide these terms by $\sqrt{3}$.

In the above analysis, it has been assumed that E is not dependent on small variations in the test temperature. This assumption is generally valid for most tests on metallic alloys when the variations in the test temperature are small (typically $\leq\pm 10^{o}$C). Otherwise, the temperature dependency of E must be included in the analysis.

Since the extensometer is balanced before each modulus measurement, the uncertainty in E should be insensitive to any bending that may be present in the specimen. Table A1 shows a typical uncertainty budget sheet for calculating the standard and expanded uncertainties for E.

Table A1 A Typical Uncertainty Budget Worksheet for Calculating the Uncertainty in Young's Modulus.

Source of uncertainty	Symbol	Value	Probability distribution	Divisor d_v	c_i	u_E	n_i or n_{eff}
Stress [1)]	s		Normal		$\frac{1}{\varepsilon}$		∞
Strain [2)]	e		Normal		$-\frac{\sigma}{\varepsilon^2}$		∞
Repeatability of measurement [3)]	u_{Erep}		Normal		1		n-1
Combined standard uncertainty	u_c		Normal			$u_c(E)$	n_{eff}
Expanded uncertainty	U		Normal				n_{eff}

1) Using Eq. (A13) and a validated calibration certificate.
2) Using Eq. (A19) and a validated calibration certificate.
3) $u_{Erep} = \frac{s}{\sqrt{n}}$, where n is the number of measurements. For single-test calculations, this source is obviously not relevant

A5. Uncertainty in the Plastic Strain Range

Current standard practices use different methods for determining the plastic strain range component and this will have an impact on the uncertainty calculation [3]. In tests conducted according to either ISO/DIS 12106 or BS 7270 procedures the uncertainty in plastic strain range can be assumed to be approximately equal to the uncertainty in the total strain range i.e. $u_{Dep} = u_{Det}$.

However, the ASTM E606-92 defines the plastic strain range as:

$$\Delta\varepsilon_p = \Delta\varepsilon_t - \frac{\Delta\sigma}{E} \qquad \text{(A26)}$$

The sensitivity coefficients c_i associated with the uncertainties in De_t, Ds and E are:

$$\frac{\partial\Delta\varepsilon_p}{\partial\Delta\varepsilon t} = 1 \qquad \text{(A27)}$$

$$\frac{\partial\Delta\varepsilon_p}{\partial\Delta\sigma} = -\frac{1}{E} \qquad \text{(A28)}$$

and

$$\frac{\partial\Delta\varepsilon_p}{E} = \frac{\Delta\sigma}{E^2} \qquad \text{(A29)}$$

The uncertainty u_E is:

$$u_{\Delta\varepsilon p} = \sqrt{u_{\Delta\varepsilon t}^2 - \left(\frac{1}{E}\right)^2 u_{\Delta\sigma}^2 + \left(\frac{\Delta\sigma}{E^2}\right)^2 u_E^2} \qquad \text{(A30)}$$

In Eq. (A30), it is assumed that each of the uncertainties u_{De_t}, u_S and u_E have a normal probability distribution. For rectangular distributions divide these terms by $\sqrt{3}$.

Table A2 shows a typical uncertainty budget sheet for calculating the uncertainty in $\Delta\varepsilon_p$ when the ASTM E606 definition is used.

Table A2 A Typical Uncertainty Budget Worksheet for Calculating the Uncertainty in $\Delta\varepsilon_p$ (ASTM definition).

Source of uncertainty	Symbol	Value	Probability distribution	Divisor d_v	c_i	$u(De_p)$	n_i or n_{eff}
Total strain range	De_t		Rectangular	√3	1		∞
Stress range	Ds		Rectangular	√3	$-\frac{1}{E}$		∞
Young's modulus	E		Rectangular	√3	$\frac{\Delta\sigma}{E^2}$		∞
Combined standard uncertainty	u_c		Normal			$u_c(De_p)$	
Expanded uncertainty	U		Normal (k=2)				

A6. Uncertainty in N_f due to Specimen Bending

Specimen bending arises from misalignments between the axes of the specimen and the grips. There are two sources of misalignment, the first is in the form of angular and/or lateral displacements of the grip axes relative to each other. The second source arises from the specimen itself due to geometrical non-conformance of the parts that influence its alignment with the grips (i.e. departure from ideal parallelism, concentricity, roundness, perpendicularity, etc.) Specimen bending is also affected by the ratio of the stiffness of the specimen and the stiffness of the test machine - the lower the specimen stiffness, the higher the bending. Specimen percent bending is also dependent on the applied force.

The uncertainty in fatigue life due to specimen bending is calculated according to the formula [5]:

$$u_b = c_i \psi N_f \qquad \text{(A31)}$$

where $c_i = \frac{1}{\alpha}$, ***a*** is the slope of the tangent to the log De_t versus log N_f curve and ***y*** is the error in De_t due to specimen bending.

To determine α, a best fit curve should be established by computer using an appropriate curve fitting routine. The value of α corresponding to a given value of $\Delta\varepsilon_t$ or N_f can then be determined from the differentiation of the mathematical relationship or manually from the curve itself (using sharp, thin lines and with the plot expanded on an A3 or A4 sheet). Care should be taken to determine α as accurately as possible to reduce consequent errors in the uncertainty calculations.

Figure A1 shows an example for Nimonic 101 at 850°C. The fatigue life as a function of the total strain range may be reasonably represented by the polynomial equation:

$$\log \Delta\varepsilon_t = 1.22344 - 0.5942 \log N_f + 0.05511 \log N_f{}^2 \qquad \text{(A32)}$$

and α is obtained from differentiating the above equation thus:

$$\alpha = \frac{d(\log \Delta\varepsilon_t)}{d(\log N_f)} = -0.5942 + 0.11 \log N_f \qquad \text{(A33)}$$

It should be noted that in Eq. (A31), the fatigue life will always be reduced due to specimen bending because α is negative and γ positive.

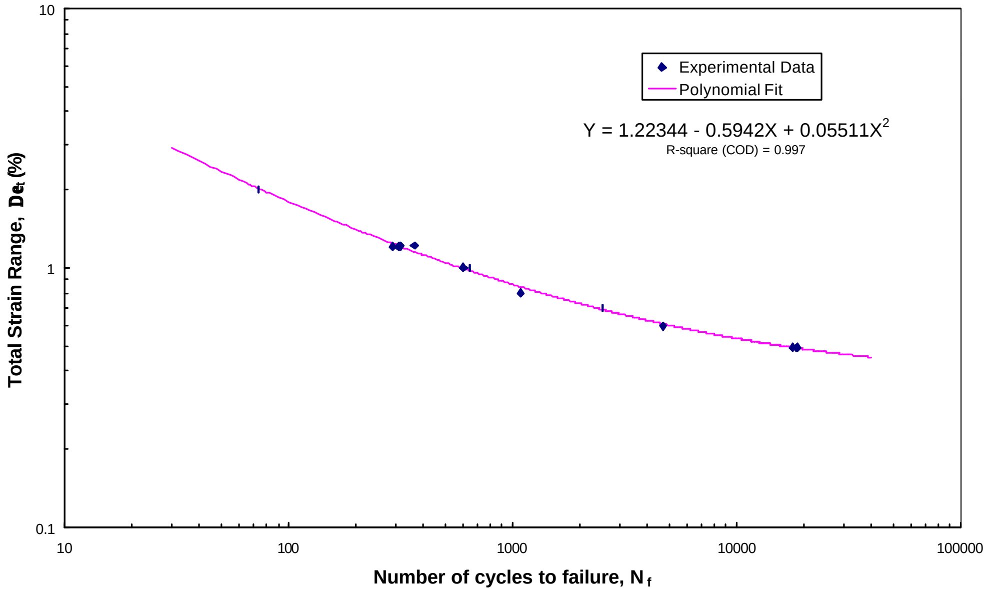

Fig. A1 Fatigue life as a function of $\Delta\varepsilon_t$ for Nimonic 101 at 850°C.

Reference 4 describes a procedure for determining the Bending Reversibility Parameter, γ, needed for the present calculations. Figure A2 shows a typical example of these results.

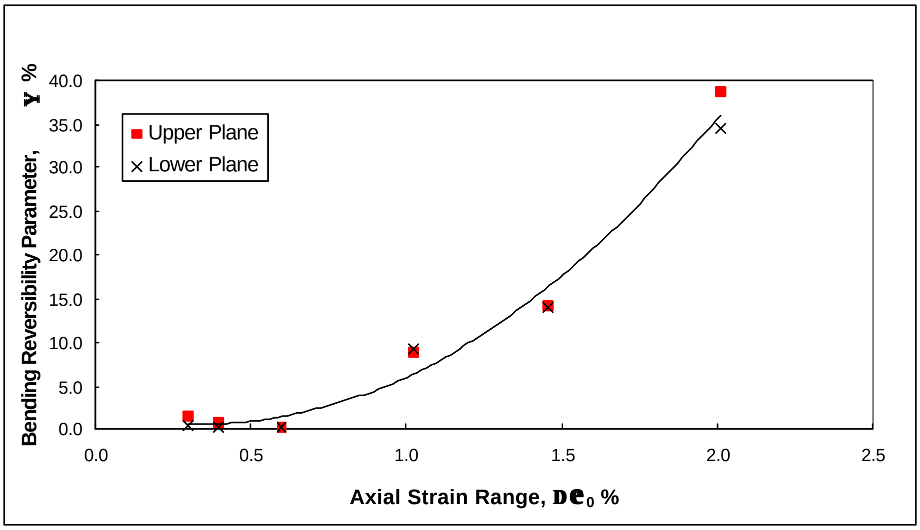

Fig. A2 Plastic bending measurements on Nimonic 101 specimens using two sets of strain gauges in accordance with Ref. [4].

A7. Uncertainty in N_f due to Errors in Strain Measurement

Similarly, the uncertainty in fatigue life due to errors in strain measurement can be expressed as:

$$u(N_f)_{\Delta\varepsilon} = c_i \delta_{\Delta\varepsilon} N_f \qquad \text{(A34)}$$

where $c_i = \frac{1}{\alpha}$, α is the slope of the tangent to the log $\Delta\varepsilon_t$ versus log N_f curve and $\delta_{\Delta\varepsilon}$ is the error in the measured total strain range. This is equal to the sum of the estimated standard uncertainty in measuring the extension, which should be obtained from a valid extensometer calibration certificate, plus the error in the gauge length (due to resetting the indicated extensometer reading at the beginning of each fatigue test.) Note that the extensometer performance may depend on its temperature stability. If the extensometer temperature fluctuates significantly (i.e. more than ±5°C), then this additional contributing factor should be considered. To determine α, see Section A5.

A8. Uncertainty in N_f due to Errors in the Temperature Measurement

The uncertainty in fatigue life due to errors in the temperature measurement and control may be expressed as [5]:

$$u(N_f)_T = c_T \delta_T \qquad \text{(A35)}$$

where δ_T is the root sum squares of the error limits of all sources contributing to the temperature measurement and control (i.e. δ_{Tc}, δ_{Ts}, δ_{Tt}, and δ_{Tu}).

The sensitivity coefficient c_T may be determined from LCF tests conducted over a range of temperatures about the nominal temperature for which the uncertainty is being estimated [5]. Figure A3 shows an example of LCF test results for Nimonic 101 at three different temperatures. In this example, the nominal test temperature (in the test programme) was 850°C and the other 2 temperatures selected were 25°C higher or lower than this (being 5 times the allowable error tolerance of 5°C).

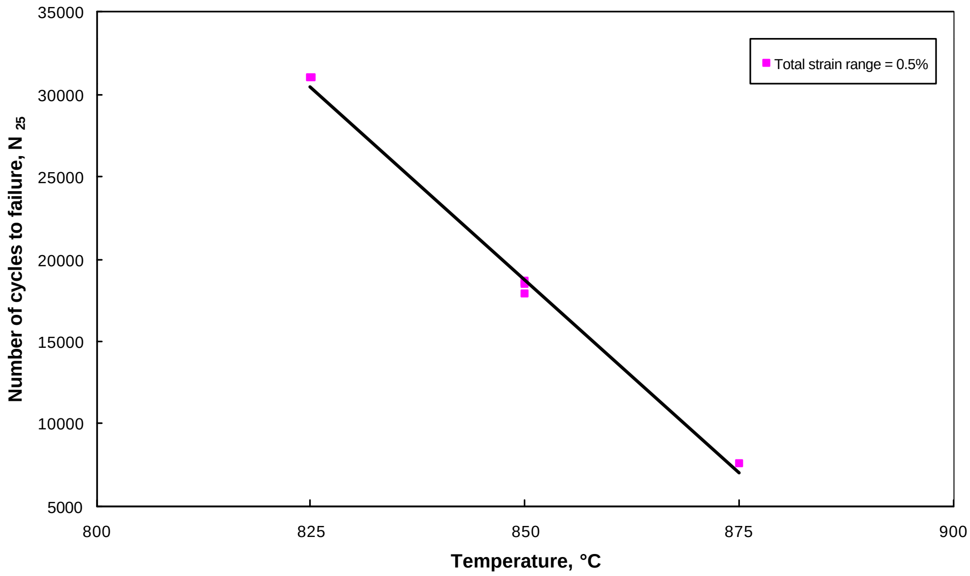

Fig. A3 Variation in fatigue life of Nimonic 101 as a function of test temperature.

A9. Uncertainty due to the Method of Determining N_f

Complete fracture of the specimen into two parts provides a unique number of cycles with 100% certainty, providing the cycle counter records individual cycle numbers (not 10s or 100s). However, it is common practice in LCF testing to define failure according to the ability of the specimen to sustain a certain level of tensile force. Failure is often defined as the point at which the maximum force (or the associated elastic modulus, E_1, measured when unloading from a peak tensile stress - see Fig.1) decreases by approximately 50%. Other values such as 25%, 10%, 5% and 2% have also been used. The lower values (<25%) are usually determined retrospectively after the test has finished and would be expected to have a higher uncertainty. The uncertainty here is associated purely with the method of determining N_f (i.e. manually from graphs of tensile stress versus the number of cycles or by computer) and not due to the chosen failure criterion, although it can be affected by the latter.

Estimating the uncertainty due to the method of determining N_f is very much dependent on the material behaviour and the exact method used.

A10. Repeatability of N_f

Repeatability of N_f i.e. uncertainty in its mean value is a *Type A* uncertainty contribution. It is the standard deviation of the estimated mean value of a series of test results under the same conditions considered in the uncertainty analysis as follows:

$$u(N_f)_{rep} = \frac{1}{\sqrt{n}}\left[\sqrt{\frac{1}{n-1}\sum_{1}^{n}\left(N_f - \overline{N_f}\right)^2}\right] \tag{A36}$$

Where n is the number of tests and $\overline{N_f}$ is the mean number of cycles to failure. Obviously, if only one test is carried out then this source of uncertainty is not relevant.

APPENDIX B

A WORKED EXAMPLE FOR CALCULATING UNCERTAINTIES IN LCF TEST RESULTS FROM A SERIES OF TESTS AT AN ELEVATED TEMPERATURE

B1. Introduction

A customer asked a test laboratory to carry out a series of strain-controlled LCF tests at a nominal total strain range, $\Delta\varepsilon_t$, of 0.5% and a nominal test temperature of 850^{o}C. The tests were carried out according to ISO\DIS 12106 on straight-sided cylindrical specimens (8 mm in diameter and with a 16 mm parallel length) made of superalloy Nimonic 101 material. The laboratory considered the sources of uncertainty in its test facility and found that the sources of uncertainty in fatigue life test results were identical to those described in Table 2 of the Main Procedure.

B2. Estimation of Input Quantities to The Uncertainty Analysis

1 All tests were carried out according to the laboratory's own written procedure using an appropriately calibrated fatigue test facility. The test facility was located in a temperature-controlled environment (21±2^{o}C).

2 The diameter of each specimen was measured using a calibrated digital micrometer with an accuracy of ± 0.002 mm (manufacturer's specification) and a resolution of ±0.001 mm. Five readings were taken, three at 120 degree intervals at the centre of the specimen and two readings at locations near the ends of its parallel length.

3 The tests were carried out on a servo-electric machine under total strain controlled conditions using a triangular strain wave with a strain ratio of -1 and a strain rate of 1.0 x10^{-3} s^{-1} (= 6.0 %/minute). The machine was calibrated to Class 1.0 according to ISO 7500/1.

4 The axial strain was measured using a single-sided extensometer with a nominal gauge length of 12.0 mm. The extensometer complied with Class 0.5 specification according to EN 10002-4:1994.

5 The error in the extensometer gauge length (due to resetting of the indicated extensometer reading at the beginning of each fatigue test) was estimated to be within ± 0.12 mm, which is equivalent to ± 1.0% of the nominal gauge length.

6 Specimen bending measurements were carried out on a strain-gauged specimen at ambient temperature in accordance with Procedure B in Ref. [3]. Figure A2 shows the results of the measurement.

7 The specimen temperature was measured using two type R thermocouples that were calibrated against a standard reference thermocouple. The calibration results

showed that, at a nominal temperature of 850°C, the error in the indicated thermocouple reading was within ±0.5°C.

8 The temperature variability in the gauge section was measured using a dummy specimen with three thermocouples attached to the specimen surface, one at the centre and one at each end of the gauge length. The temperature variability was found to be within ±1.4°C.

9 During each test the temperature was recorded at regular intervals from which the indicated thermocouple readings were maintained during the test to within ±2.0°C.

10 Values of the temperature sensitivity coefficient, c_T, were determined from LCF tests carried out at nominal temperatures of 825°C and 875°C. The results are shown graphically in Fig. A3.

11 The number of cycles to failure was determined by computer when the maximum stress dropped by 25% from the value at mid-life. It is estimated that the resultant values contain an error of 2%.

12 Three or four repeat fatigue tests were carried out at each pre-selected value of $\Delta\varepsilon_t$.

B3. Uncertainty in the Stress Values

Input values:

d = 8.000 mm (nominal value)

Assuming a value of $u_d = \pm\, 0.01$ mm, then $\frac{u_d}{d} = \pm 0.125\%$

$$u_F = \frac{U_F}{k} = \frac{\pm\, 0.44\%}{2} F$$ (See Table 1 in Section 3 of the *Manual [1]*)

$$\frac{u_F}{F} = \pm 0.22\%$$

Substituting the above quantities into Eqs. (A4) and (A14) gives the following estimates of uncertainties in the cross-sectional area and the stress:

$$\frac{u_{Ao}}{A_o} = \pm 0.25\%$$

$$\frac{u_\sigma}{\sigma} = \pm\, 0.33\%$$

B4. Uncertainty in the Strain Values

Input values:

Class 0.5 extensometer

L_o = 12.0 mm

u_e = ± 0.75 microns, or $\frac{u_e}{e}$ = ± 0.5% (whichever is greater, see Table 2 in Section 3 of the *Manual* [1])

$\frac{u_{Lo}}{L_o}$ = ± 1.0% (due to resetting the extensometer)

From Eq. (A20), the estimated relative uncertainty in strain is:

$\frac{u_\varepsilon}{\varepsilon}$ = ± 1.12% (for e ≥ 0.3 mm)

$$\frac{u_\varepsilon}{\varepsilon} = \pm\sqrt{\left(\frac{0.00075}{e}\right)^2 + 0.01^2}$$ (for e ≤ 0.3 mm)

B4. Uncertainty in Young's Modulus

In the current tests, Young's modulus measurements were performed at a stress level of typically 500 MPa. The corresponding extension was 27.2 microns. It should be noted that in Young's modulus measurement, the relative rather than absolute value of the strain are relevant and the most important components of uncertainty in measuring the extension are those relating to the resolution and linearity of performance of the extensometer.

Input values:

Average E value = 220.84 GPa (34 tests, s = 6.99 GPa)
s = 500 MPa
e = 2.26 10^{-3}
u_s = 1.65 MPa (being = 0.33% σ)
u_e = ±0.5 micron (estimated value)
u_e = ±4.73 10^{-5}

Table B1 Uncertainty Budget Calculations For Young's Modulus.

Source of uncertainty	Symbol	Value ±	Probability distribution	Divisor d_v	c_i	u_E ±MPa	n_i or n_{eff}
Stress	s	1.65 MPa	Normal	1	$\frac{1}{\varepsilon}$	730	∞
Strain	e	$4.73\ 10^{-5}$	Normal	1	$-\frac{\sigma}{\varepsilon^2}$	4630	∞
Repeatability of measurement [1)]	u_{Erep}		Normal		1	1199	33
Combined standard uncertainty	u_c		Normal			4838	
Expanded uncertainty	U		Normal (k=2)			9676	

1) $u_{Erep} = \frac{s}{\sqrt{n}}$, where n =34 is the number of measurements.

<u>Reported Result</u>

The estimated value of Young's modulus is 220.84 ± 9.68 GPa

The above reported expanded uncertainty is based on a standard uncertainty multiplied by a coverage factor k=2,which provides a level of confidence of approximately 95 percent. The uncertainty evaluation was carried out in accordance with UNCERT CoP 02: 2000.

B5. Uncertainty in N_f

Table B2 lists the input quantities used to produce the uncertainty budget shown in Table B3.

Table B2 Input quantities used for producing Table B3.

Quantity	Symbol	Values	Mean	standard deviation
Total strain range, %	De_t	0.492, 0.495, 0.495	0.494	0.0017
Number of cycles to failure	N_f	18481, 18643, 17912	18345	384
Parameter [1]	a	-0.12		
Bending Reversibility Parameter[2]	y	1.0%		
Temperature sensitivity coeft[3]	c_T	-473 cycles/^{o}C		
Error in extension measurement	d_x	±0.5%		
Error in gauge length	d_{gl}	±1.0%		
Error in temperature measurement (thermocouple)	d_{Tm}	±0.5 ^{o}C		
Error in temperature uniformity	d_{Tu}	±1.4 ^{o}C		
Error in temperature control	d_{Tc}	±2.0 ^{o}C		

1) α is obtained from differentiating the equation shown in Fig. A1 thus

$$\alpha = \frac{d(\log\Delta\varepsilon_t)}{d(\log N_f)} = -0.5942 + 2\times0.5511\times\log N_f$$

2) From Fig. A2.

3) c_T = slope of the straight line shown in Fig. A3.

Table B3 Uncertainty Budget for Calculating the Uncertainty in LCF Life in a Series of Identical Strain-Controlled Tests ($\Delta\varepsilon_t$ = 0.5% & 850^{o}C).

Source of uncertainty	Symbol	Value	Probability distribution	Divisor d_v	c_i	$u_i(N_f)$ ± cycles	ν_i or ν_{eff}
Specimen bending [1]	y	+1.0%	Rectangular	√3	-8.33	882	∞
Strain measurement [2]	De	±1.5%	Rectangular	√3	-8.33	1323	∞
Temperature measurement [3]	T	±2.5^{o}C	Rectangular	√3	-473	683	∞
Method of determining N_f [4]	$u(N_f)_{det}$	±2.0%	Rectangular	√3	1.0	212	∞
Repeatability of N_f [5]	$u(N_f)_{rep}$	±222 cycles	Normal	1.0	1.0	222	2
Combined standard uncertainty	u_c		Normal			1758	>100[6]
Expanded uncertainty	U		Normal (k = 2) [7]			3515	>100

1) Value of $c_i = (1/a) = (1 / -0.12)$ and
$u(N_f) = \pm0.01\times(1/\sqrt{3})\times8.33\times18{,}345$ cycles.

2) Estimated value = 0.5 % (for Class 0.5 extensometer) + 1.0% (due to uncertainty in the gauge length).

3) Estimated value of e_T = $\sqrt{0.5^2 + 1.4^2 + 2.0^2}$ = 2.49

4) Values of N_f were determined by computer and it was estimated that these values contain an error of 2%.

5) The standard uncertainty was calculated according to Eq. (A36) [= the standard deviation, *s*, shown in Table B2 divided by √3, where 3 is the number of tests].

6) The effective degrees of freedom, n_{eff}, were calculated according to Eq. (7) in *Manual* [1], Section 2, viz.:

$$\nu_{eff} = \frac{1758^4}{0 + 0 + 0 + 0 + \frac{222^4}{2}}$$

7) A coverage factor k = 2 was obtained from the student's t-distribution table included in the *Manual* [1], Section 2.

Reported Result

The number of cycles to failure in the test series is 18345 ± 3515 cycles.

The above reported expanded uncertainty is based on a standard uncertainty multiplied by a coverage factor k=2, which for a t-distribution with an effective degrees of freedom, n_{eff}>100, corresponds to a level of confidence of approximately 95 percent. The uncertainty evaluation was carried out in accordance with UNCERT CoP 02: 2000.

Notes:

(1) The uncertainty calculations exclude residual stress effects.
(2) Uncertainties in the parameters a and Y have not been included.